THE MATHEMATICAL GAZETTE

EDITED BY
W. J. GREENSTREET, M.A.

WITH THE CO-OPERATION OF
F. S. MACAULAY, M.A., D.Sc.
AND
PROF. E. T. WHITTAKER, M.A., F.R.S.

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NOTICE.

The following Reports have been issued by the Association:—(i) "Revised Report on the Teaching of Elementary Algebra and Numerical Trigonometry" (1911), price 3d. net; (ii) "Report on the Correlation of Mathematical and Science Teaching," by a Joint Committee of the Mathematical Association and the Association of Public School Science Masters, price 6d. net; (iii) A General Mathematical Syllabus for Non-Specialists in Public Schools, price 2d. net. These reports may be obtained from the Publishers of the *Gazette*.

(iv) **Catalogue of current Mathematical Journals**, with the names of the Libraries where they may be found. Pp. 40. Price, 2s. 6d. net.

"Even a superficial study convinces the reader of the general completeness of the catalogue, and of the marvellous care and labour which have gone to its compilation."—*Science Progress*, Jan. 1916.

(v) **Report of the Girls' Schools Committee, 1916**: Elementary Mathematics in Girls' Schools. Pp. 26. 1s. net.

(vi) **Report on the Teaching of Mechanics, 1918** (*Mathematical Gazette*, No. 137). 1s. 6d. net.

(vii) **Report on the Teaching of Mathematics in Public and Secondary Schools, 1919** (*Mathematical Gazette*, No. 143). 2s. net.

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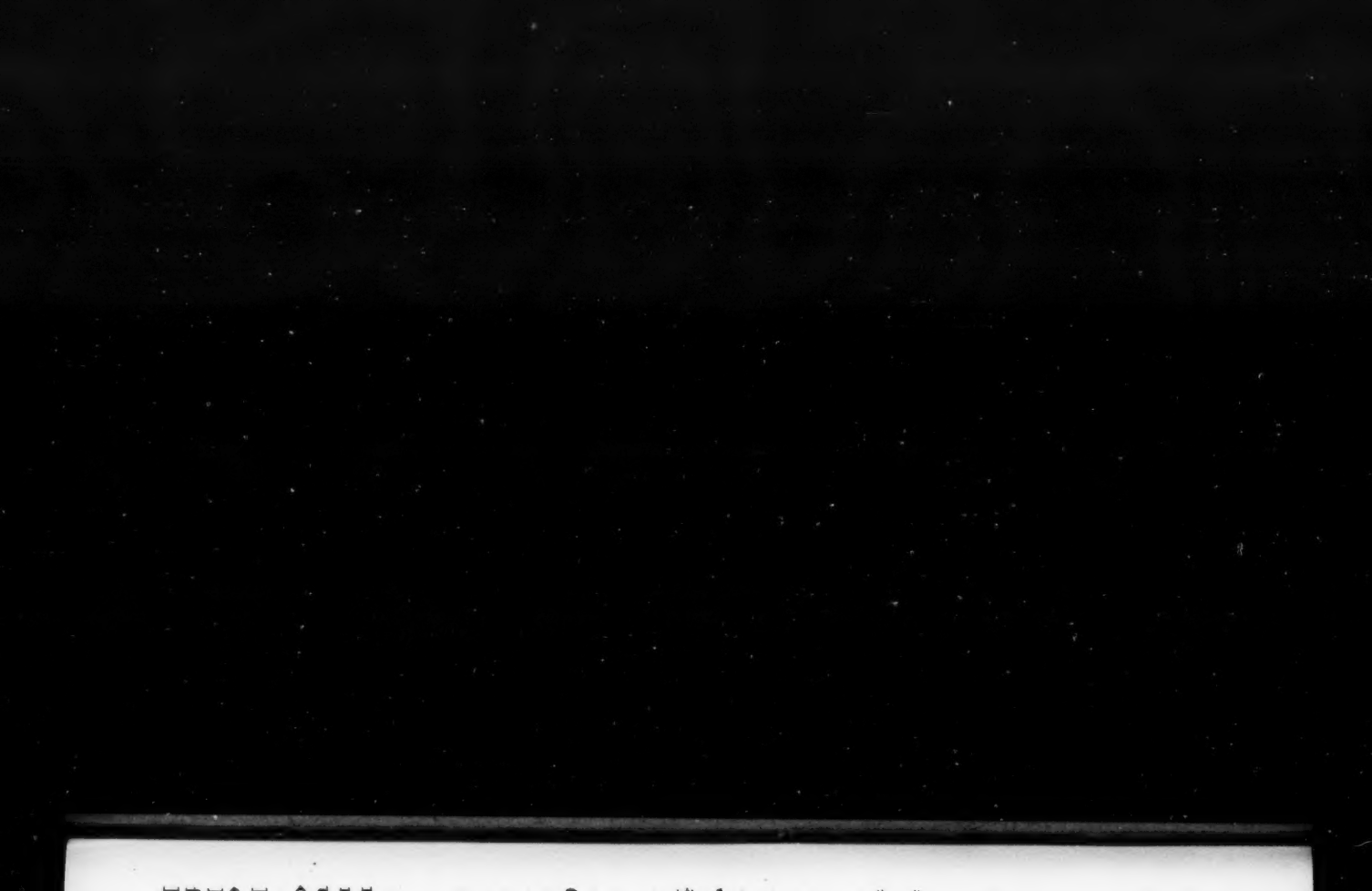

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VOL. X. MARCH, 1921. No. 151.

The Mathematical Association.

President: PROFESSOR E. T. WHITTAKER, M.A., Sc.D., F.R.S.

THE Annual Meeting of the Mathematical Association was held at the London Day Training College, Southampton Row, London, W.C. 1, on Tuesday, 4th January, 1921, at 10 a.m. and 2.30 p.m.

10 *a.m.*

BUSINESS.

(1) The following Report of the Council for the year 1920 was distributed and taken as read. The proposal by the Council that the Annual Subscription paid to the Association by a Local Branch be increased from 10s. to 15s. to bring it into line with the ordinary member's subscription, was incorporated in the Report, which was then adopted.

REPORT OF THE COUNCIL FOR THE YEAR 1920.

During the year 1920, 94 new members have been elected. The number of members now on the Roll is 760. Of these, 8 are honorary members, 46 are life members by composition, 30 are life members under the old rule, and 676 are ordinary members. The number of associates is about 225.

The Council regret to have to record the deaths of Professor John Perry, F.R.S., Mr. A. Brand, Mr. H. D. Ellis, the Rev. J. A. C. Lane of Uppingham, Mr. H. G. Pearsall of Batley, and Dr. T. B. Sprague. Mr. Ellis was one of the Honorary Secretaries of the Association from 1900 to 1915. Professor Perry was an honorary member. Dr. T. B. Sprague had been a member of the Association since 1873.

He was Senior Wrangler in 1853, and died in November, 1920, in his ninety-first year.

The first summer meeting of the Association was held at Leeds on 31st May and 1st June, 1920, when the members and associates were invited to dinner by the Council and Senate of the University of Leeds. The Council of the Association desire to express their warm appreciation of the generous hospitality accorded to the Association by the Council and Senate of the University. The papers read at the meeting have been published in the *Gazette*.

An election of members to serve on the new Teaching Committees was held early in 1920, after an interval of six years. The old Committees remained in office throughout the years of war. The Council recommend that the new Committees be continued in office until the spring of 1923. At the next annual meeting of the Association the Council will propose an alteration of the scheme under which the Committees are formed, so far as concerns the manner of nomination.

The Council are pleased to report that three new branches of the Association have been formed during the year: at Leeds (for Yorkshire), at Bristol, and at Manchester. Their officers are, for Yorkshire—*President*, Professor W. P. Milne, D.Sc.; *Hon. Secretary*, the Rev. A. V. Billen, Leeds Grammar School; for Bristol—*President*, Professor H. R. Hassé, D.Sc.; *Hon. Secretary*, Mr. R. C. Fawdry, Clifton College; for Manchester—*Hon. Secretaries*, Miss W. Garner, Whalley Range High School, and Miss M. L. Tanner, Broughton High School.

The Council propose that the Annual Subscription paid to the Association by a Local Branch be increased from 10s. to 15s. to bring it into line with the ordinary member's subscription.

Professor E. T. Whittaker, M.A., Sc.D., F.R.S., retires at this meeting from the office of President. The Council, in the name of the Association, desire to express their deep sense of the valuable services which he has rendered to the Association during the past two years, and to thank him most cordially for his close personal attention to its interests. The Council have the pleasure of nominating Professor Whittaker to be a Vice-President of the Association.

The first recorded meeting of the *Association for the Improvement of Geometrical Teaching* was held at University College on the 17th January, 1871. The fiftieth anniversary of that meeting will therefore fall in the present month. Of the 61 original members of the Association, we have still on the Roll of Members the names of Mr. Rawdon Levett, the Rev. Canon Wilson, D.D., Mr. A. A. Bourne, the Rev. W. H. Laverty, the Rev. E. F. M. MacCarthy, Mr. F. E. Marshall, the Right Hon. Lord Moulton of Bank, G.B.E., K.C.B., F.R.S., and Sir Thomas Muir, C.M.G., LL.D., F.R.S.

The Council have the pleasure of nominating the Rev. Canon Wilson, D.D., to be President of the Association in succession to

Professor E. T. Whittaker. Canon Wilson was appointed to be one of the two Vice-Presidents of the Association at the time of its formation in 1871, and held that office for the following ten years. He was also one of those present at the first meeting of the Association.

Mr. G. Goodwill and Miss J. M. Lewis retire now from the Council by rotation, and are not eligible for re-election for the coming year. The members present at the Annual Meeting will be asked to nominate and elect others to fill the vacancies.

The Council again desire to acknowledge the indebtedness of the Association to Mr. Greenstreet for his services as Editor of the *Mathematical Gazette*; and to offer their thanks to the authorities of the London Day Training College for their kindness in affording accommodation for the Annual Meeting, and for the meetings of the Council which have been held during the year.

(2) Owing to unavoidable delay in the preparation of the Balance Sheet, the Treasurer's Report for 1920 was not presented.

(3) The Election of Officers and Council for the year 1921.

On the proposal of the Council the Rev. Canon J. M. Wilson, D.D., was elected President of the Association in succession to Professor E. T. Whittaker, F.R.S.

Mr. C. Godfrey and Mr. W. M. Roberts were elected members of the Council in succession to Mr. C. Goodwill and Miss J. M. Lewis, who retired by rotation. With these substitutions, the Officers and Council were re-elected.

On the conclusion of the Discussion on his paper on "Relativity," Professor Eddington was elected an Hon. Member of the Association.

PAPERS AND DISCUSSIONS.

(4) "Relativity," by Professor A. S. Eddington, F.R.S.

(5) "Aeroplane Mathematics," by Dr. S. Brodetsky.

(6) "The Teaching of Mathematics to Boys whose chief interests are non-mathematical," by the Rev. S. H. Clarke, B.A.

TUESDAY AFTERNOON, 2.30.

(1) "Some Unsolved Questions and Topics for Research," by Professor E. T. Whittaker, Sc.D., F.R.S.

(2) The new President then took the chair and addressed the meeting on the early history of the Association.

The 17th of January, 1921, was the 50th Anniversary of the first recorded meeting of the Association.

(3) "Results of visits paid to Lycées of Paris and other centres, and the Study of Education there, particularly from the point of view of Mathematics," by Miss E. M. Read, B.Sc.

ADDRESS ON RELATIVITY.

By A. S. Eddington.

It is a familiar principle in elementary mechanics that a uniform motion of the whole system under consideration makes no difference to the phenomena, and may be ignored. For example, in calculating the motions of the planets round the sun, we do not need to pay any attention to the fact that the whole solar system is travelling towards the constellation Lyra. It does not seem to us that there is anything surprising or needing explanation in this principle; in fact, when we try to examine the idea of motion through an empty space without fixed landmarks, we are conscious of something illusory in the conception.

Einstein's principle of relativity of 1905 reiterates and emphasises this principle—that uniform motion of the whole system can be entirely ignored. But it arouses opposition because, since Newton's time, we have filled space with an aether which is the seat of all electrical and optical phenomena. The "whole system" is now the system of material bodies plus the aether; and it would seem that we can no longer give an arbitrary uniform motion to the whole system, because, whatever we do to the material bodies, the aether will slip through the interstices and remain as it was. In short, it is claimed that the aether provides the fixed landmarks which were lacking before. It is no longer a question of travelling through void—a conception to which it is difficult to attach any meaning—but of travelling through an ocean of aether.

But now comes the bombshell of experiment, to shatter the position gained by this nineteenth century offensive. Simply, as a matter of hard empirical fact, the aether does *not* provide the fixed landmarks that we imagined. The experimental test (the Michelson-Morley experiment and others) is one of the most interesting parts of this subject; but I have not time to enter into a discussion of it. It is sufficient to say that these experiments quite unexpectedly refused to give an answer to the question, what is our motion through the aether? If aether-landmarks exist, there is no known way of finding them out.

Perhaps we ought not to build too much on the experiments alone. You cannot very well prove a universal negative by experiment, or feel certain that your successors will always fail to find this aether-motion. But the experiments suggested a re-examination of the theory. Fortunately there is no disagreement about the theoretical laws of the aether; they were laid down many years ago by Maxwell in his famous set of equations. From time to time, in solving a mathematical equation, one meets with a rather annoying experience. The reduction and simplification goes on excellently; various complicated terms cancel out; and at length, just as you think you are going to arrive at the desired value of x, behold! everything disappears, and you are left with the profound but irritating truth that $0=0$. It is always like that with velocity through the aether. You form the equations relating to some actual or hypothetical experiment, and v the velocity of the apparatus through the aether duly appears in them. Then you eliminate the various unknowns, trying to isolate v. But it is a fiasco; v disappears as well, and steadily refuses to be equal to anything in particular. It is capable of strict mathematical proof that Maxwell's equations are of such a form that this must always be the result.

You will see then that both theory and experiment agree that motion through the aether is undetectable. Velocity through the aether is like that elusive personage Mrs. Harris; and Einstein has inspired us with a daring scepticism—"I don't believe there's no sich a person." It is this disbelief *

* Note that we do not reject the aether, but only velocity with respect to the aether; that is to say, the nature of the aether is such that it does not provide a standard with respect to which a velocity can be measured.

which starts us on the way to revolutionise the physical ideas of space and time.

Now, if I may pursue the simile, Mrs. Harris's adherents are rather indignant with us; it is too bad to say the lady is a myth merely because they are unable to produce her. They are determined to bring her in somehow, and they point with justifiable pride to the scheme suggested by FitzGerald and elaborated by Larmor and Lorentz, which supposes that there is a definite velocity of the earth through the aether, but compensations have been arranged by nature so that under no circumstances can this velocity ever make any difference to anything. You probably know the kind of compensations required—contraction of all standards of length, change of rate of a moving clock, increase of mass with velocity, etc. Now let us frankly admit that this scheme of nature is a tenable one. But to the relativist, its fault is that it is like the schemes of the White Knight; you remember the verse:

> But I was thinking of a plan
> To dye one's whiskers green,
> And always use so large a fan
> That they should not be seen.

Why go out of your way to introduce this hypothetical thing, velocity through the aether, and then make elaborate arrangements to keep it out of sight? Of course, in the old days when the aether was regarded as a rarefied kind of matter, it was necessary to suppose that it must have some definite velocity. But few people nowadays believe that the aether is material in the ordinary sense; and all reason for imagining a velocity relative to it as a fixed framework has disappeared.

Suppose that far away in space we had an exact replica of the solar system, travelling uniformly at a thousand miles a second relative to the actual solar system. Newtonian dynamics tells us that the motion makes no difference—that the replica would remain perfect, and the same sequence of eclipses, occultations, conjunctions, etc., would take place in it as with us. In fact, the same Nautical Almanac would serve for both. The experimental and theoretical results that we have been discussing, confirm this; and they tell us that this is true not only for the mechanical part of the phenomena, but also for electrical and optical phenomena. In the behaviour of the system there is nothing to give away that thousand miles a second velocity; it could only be detected by observing some material landmark, *e.g.* another star outside the system.

Now any point x, y, z, in the original sytem at an instant t, has an exact counterpart x', y', z' in the replica at an instant t'. You know that in ordinary mechanics there is a relation between these corresponding points and instants, viz.,

$$x' = x + ut, \quad y' = y, \quad z' = z, \quad t' = t,$$

where u is the velocity of the replica, supposed along Ox.

We now find that these are only approximate formulae, and the true relations are

$$x' = \beta(x + ut), \quad y' = y, \quad z' = z, \quad t' = \beta(t + ux/c^2),$$

where c is the velocity of light, and $\beta = (1 - u^2/c^2)^{-\frac{1}{2}}$.

These relations mix up space and time in a remarkable way. Instead of $t = t'$, *i.e.* one uniform absolute time respected by all systems, we have

$$t' = \beta(t + ux/c^2),$$

i.e. a different time for the moving system, related not only to time but also to space in the fixed system. Let us be quite clear what we mean by time in the two systems. We set an astronomer in each system to determine by means of his telescope, Nautical Almanac, etc., the time of an event visible to both. Our own astronomer finds the time to be t, the astronomer in the other system finds it to be t'; t is not equal to t', nor do they differ merely in the zero from which time is reckoned, but are connected by the complicated relation given above.

We are now ready for the next step. We see that, instead of our time-reckoning being unique and universal, the inhabitant of another star would disregard it and employ an altogether different reckoning. The present instant "NOW!" seems to us to run through space, like a clean section through the changing world, marking an instantaneous state. The changing world is thus analysed into a succession of instantaneous states. I want to persuade you that this is nothing more than a mathematical analysis—sometimes useful, sometimes mischievous—it is not the natural taking to pieces of a puzzle, but a rude hacking through with the carving knife. The instant "Now," so far as I am aware of it, exists just where I am; there is no reason to picture it as extending into other parts of space where I am not.* It is true that if I am excited watching a football match, I may imagine myself among the players, carrying my "Now" with me; but that is poetry, not science. Cool science tells me that what I see happen now *did not happen now*; the impression has travelled to me with a finite velocity. Science—even of the most old-fashioned type—will have none of my instinctive extension of the instant "Now"; in place of it it has constructed an extension defined according to mathematical rules, which gives us the world-wide instant used in astronomy, physics, and to some extent in ordinary affairs of life. But this agreed continuation is a mathematical construction, fixed up by a caucus of astronomers, who agreed chiefly because they had not sufficient imagination to differ. When I say that it is agreed, I can answer only for terrestrial astronomers; the astronomers on that replica of the solar system we have been speaking of will profoundly disagree. They (if they are replicas of astronomers here) have adopted a different continuation of the instant "Now"—because of their different motion. Two events, which we call simultaneous, may be 100 years apart in their reckoning.

Our world-wide instant "Now" is in no way better than theirs; it is a despicable mathematical fiction convenient for us, as theirs is for them. To say that the world consists of a succession of instantaneous states means just as much and as little as to say that a piece of bacon consists of rashers. The rashers are not predetermined, and until the knife has been used we can only contemplate the solid block of bacon. The rashers can be cut in different directions, and so too the different observers cut the changing world in different directions to obtain the slices which they call instantaneous states. Until we fix the observer who is to make the dissection, we can only contemplate the four-dimensional block of undivided space-time.

I can only touch on one or two outstanding points, and will now pass on to what is called General Relativity. We progress by considering how nature would appear regarded from the point of view of some one differently, and perhaps better, situated than ourselves. Thus Copernicus revolutionised astronomy by considering how the solar system would appear seen from the sun. We have just now reached the four-dimensional world of space-time, by considering the point of view of observers in very rapid motion relative to us. For the next step, I must ask you to adopt the point of view of a man who has fallen out of an aeroplane.

Now, of course, it is easy to say that that is a ridiculous and unnatural situation to contemplate the world from, and to treat it as a joke. What a humorous old fellow Copernicus was when he told us to take the point of

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* Some complication is introduced by the fact that the word "Now" may refer not to an exact instant of time, but to a period of time which, though short, is not infinitesimal. The instant in New York which is simultaneous with the instant here is indeterminate (except by arbitrary mathematical convention); but there are limits to the ambiguity, and we can fix on a period of about $\frac{1}{10}$ sec. in New York, which certainly overlaps the instant "Now" here. As we go further away the limits widen. Thus the lover who says to himself "She is thinking of me *now*" need not be troubled by the ambiguity, for her thought would certainly endure $\frac{1}{10}$ of a second. But if the absent one were on the planet Neptune (where the ambiguity owing to the increased distance amounts to about four hours), he would be deprived of this consolation.

view of a man sitting frizzling in the sun, instead of remaining on this solid immovable earth! Need we take him seriously? Let me explain why I think the falling aviator is better situated than we are. My body is a delicate piece of apparatus for recording observations of nature; but unfortunately the soles of my boots are being battered by the molecules of the ground all the time with a force equal to some ten stone weight. (Action and reaction, you remember, are equal and opposite.) Now, if you were using some delicate galvanometer, would you willingly allow someone to hammer on it like that? To put it mildly it would complicate things a great deal. So I propose that we should get rid of this molecular bombardment by jumping out of an aeroplane, so that for a little while we may be quite undisturbed in our investigations.

Newton contemplated the falling apple and produced his famous theory of gravitation. But had it fallen to the lot of the apple to produce the theory of gravitation, I think the result would have been something like Einstein's. At least the story goes that Einstein was inspired with the principles of his theory by talking to a man who had fallen from a third-floor window.

Let our falling aviator perform the experiment of dropping an apple which he was holding in his hand. The apple cannot fall any more than it was doing already, and remains in contact with his hand—at rest, as the aviator would say. He has no knowledge of any field of force tending to accelerate the motion of unsupported objects; he reports that things at rest remain at rest, and things in uniform motion remain in uniform motion. No doubt he notices a tree coming faster and faster to meet him; but the reason is obvious—it is not moving freely, but is being shoved up from below by molecular bombardment. For the falling aviator the field of gravitational force, which has so puzzled us, has vanished.

A new point of view is not in itself a new theory, though it may lead to one; but, when we are seeking to understand the nature of gravitation, it is something to have shown that two people may disagree as to whether in any region there exists a gravitational field to be explained. I am not so much suggesting that the gravitational field is an illusion, as that it is slippery; and in any picture we try to form, we must somehow recognise this slippery character. Suppose that, instead of watching things close at hand, the aviator looks further afield. If he looks through the earth to the antipodes, he finds free bodies there are falling towards him, not merely at 32 feet/sec^2., but at 64 feet/sec^2. By his own fall he has abolished gravitation near him, only to pile it up elsewhere—as though he had smoothed out a pucker at one point and it had run off somewhere else, as puckers do. May we not picture the field as a pucker in the world? You may say that the pucker is only an analogy; but, after all, what is any description we can give of the things around us but an analogy of those inscrutable processes of nature, which necessarily must transcend the pictures of our senses? Our old picture of a force on the falling body—something tugging at it—is only an anthropomorphic analogy. If on examination the pucker is a good analogy, if we can discover no point where the analogy breaks down, if we can press it even further than our old analogy of a field of force, so that it explains things which the force does not—then it seems to me we are bound to remodel our theory of nature according to the new outlook which it gives.

The theory of non-Euclidean or puckered space-time has all these advantages. Needless to say I cannot here give any account of that wonderful mathematical analysis by which Einstein followed up the initial idea. I venture to say that there has never been anything to equal it in the history of mathematical physics.

In Euclidean geometry the fundamental locus is the straight line. In non-Euclidean geometry the corresponding fundamental locus is called the geodesic—it is the analogue of the straight line, but you must not call it a straight line,

because it has not the familiar properties of one. We can construct a non-Euclidean geometry associated with the solar system which has the following property—that the tracks of the planets round the sun (both the planets which exist, and planets which might, but do not, exist) are the geodesics of the new geometry. This geometry is four-dimensional, so that it gives correctly not only the tracks of the planets through space, but their times in these tracks. This non-Euclidean geometry then is a way of describing the sun's gravitational field; I do not at the moment commit myself to saying that it is the geometry of space in the solar system in the ordinary sense. I will rather speak of it as a correlated geometry—correlated so that the geodesics in this geometry coincide with the tracks of particles in the gravitational field. Now the phrase non-Euclidean is, of course, merely a negative epithet; the geometry is not Euclidean, it is of some other kind, and we must find out what kind. We can make a few broad classifications of non-Euclidean geometries, and it turns out that the geometry correlated with a gravitational field is always of one of these fundamental classes, namely, that of a space curved "no higher than the first degree." Never mind what "the first degree" means. The point is that the mathematician can recognise the geometry alluded to, and if we tell him that the geometry correlated to the region round the sun is that of a space curved no higher than the first degree, he can immediately calculate the orbit of any planet or comet just as well as from Newton's statement about the inverse square. The statement is not quite so concise as Newton's, but I think the underlying idea is almost more concise. And as regards accuracy, the two statements have as you know been put to the test, and Einstein's comes off best, with Newton's *proxime accessit*.

There are some who would like to stop at this point. I think, for example, Prof. Whitehead would say—"Einstein's method of summarising mechanical laws by describing a correlated geometry is ingenious; it may seem a little outlandish, but if you examine any advanced text-book of mechanics (Prof. Whittaker's for instance) you will find the same kind of process. It is a fashion due mainly to Hertz. But do not go mixing up this *correlated* geometry with the *actual* geometry of space in the solar system." But that is just what Einstein insists on doing, and if you stop short of this identification of the correlated and the actual geometry—the geometry of mechanics and the geometry of extension—you miss the most beautiful part of the idea. The so-called mechanical problem of observing the course of a moving particle, and the so-called geometrical problem of observing the indications of a measuring scale are not to be separated into water-tight compartments. One law can be formulated which covers all the mechanics of motion and the geometry of measurement. It is this unification which constitutes the great advance of Einstein's theory.

I have rashly undertaken to say something about the application of these new ideas to elementary mathematical teaching. Let me say at once that I have no pressing reforms to advocate nor any definite policy to urge. I am convinced that the relativity outlook will in time lose its strangeness and become a commonplace of educated thought, just as the Copernican system has done. It is clear too that the relativity outlook presents less difficulty to the younger minds than to the older, so that there is good reason for introducing it at an early stage. But its introduction into the school must be a matter of atmosphere rather than of specific teaching. It would be unwise for one who has no experience of your problems of teaching mathematics, to attempt to suggest the precise opportunities for spreading this "Bolshevist atmosphere" among youthful minds. I can only speak in general terms.

The emphasis on the practical and experimental side of geometry, which is, I suppose, now generally admitted in schools, is a help in the right direction. I sometimes think how surprised and gratified our old mathematical masters must be nowadays; how many stalwarts do we not find now defending

Euclidean geometry as a necessity of thought, who certainly did not seem to feel any overwhelming necessity of that kind in their schooldays! But I think the belief that we can prove by internal cogitation that things must necessarily be as they are, is obsolescent; and now, when we discover that cardboard triangles on the same base and between the same parallels do actually turn out to be equal in area, our attitude is that described by Einstein as of "thankful surprise"—that nature should conform to so simple a rule when things might have been much more complicated. By theoretical geometry we bind together these experimental results, just as by mathematical physics we bind together the results of experimental optics. But we seek to learn humbly from nature what actually are the laws she has chosen, whether they be of space or of light; we do not invent a code out of our heads, and call on nature to conform to it, and blame the materials she supplies when it appears to go wrong. Also we can now perceive clearly what is the subject-matter of geometry. What is the subject of the experiments on which your schoolboys are instructed to base it? Cardboard, compasses, divided scales—in short, matter of some kind. Laws of geometry are laws relating to matter,—in particular to that special property of matter known as *extension*. It is a harmless abstraction to imagine all the other qualities of matter removed, leaving only its extension; but we realise that school-geometry deals with an extension which presupposes matter. Dismiss the absurd notion that your experiments are teaching you the properties of emptiness.

Mechanics, I fear, presents difficulties. For example, we now know that the law of the parallelogram of velocities is untrue; and any argument which seeks to establish it begs the question. But I am inclined to think that that may perhaps have to be regarded as a professional secret, scrupulously guarded from those of tender years. With regard to force, mass and inertia, I am inclined to think that general relativity may even relieve difficulty. If any one is contemplating writing a text-book on elementary mechanics, I would strongly recommend him to examine the relativity idea of these things; and, if practicable, to strike out rather boldly. But the detailed application requires much careful thought. The golden rule is—when in difficulty, appeal to experiment. Do not always think it necessary to give a reason why nature conforms to certain fundamental laws. Nature no doubt has her reasons; but they will appear (if at all) in the last chapters of some profound treatise on electromagnetic theory, not in the initial pages of a school text-book.

A. S. EDDINGTON.

Observatory, Cambridge.

In the discussion that followed, Professor E. T. Whittaker, Dr. Sheppard, Mr. C. Godfrey, Col. Alan Cunningham, Mr. C. V. Durell, and Dr. Brodetsky took part.

The next paper—by Dr. Brodetsky—is printed as March *Gazette*, No. 152.

GLEANINGS FAR AND NEAR.

70. The Educational Value of Experiment.

"It may be said that the fact makes a stronger impression on the boy through the medium of his sight, that he believes it the more confidently. I say that this ought not to be the case. If he does not believe the statements of his tutor, probably a clergyman of mature knowledge, recognised ability, and blameless character—his suspicion is irrational, and manifests a want of the power of appreciating evidence, a want fatal to his success in that branch of science which he is supposed to be cultivating."—Todhunter's *The Conflict of Studies*, 1873, p. 17.

THE TEACHING OF MATHEMATICS TO BOYS WHOSE CHIEF INTERESTS ARE NON-MATHEMATICAL.

By the Rev. S. H. Clarke.

In the Report of the Committee of this Association published a year ago, it was asserted that a boy's education "should enable him not only to apply Mathematics to practical affairs, but also to have some appreciation of the greater problems of the world, the solution of which depends on Mathematics and Science. . . ." Again, "the material side of modern civilisation stands on a foundation of Applied Mathematics. This statement will not be controverted by anyone familiar with the work of modern scientists and engineers and with the part played by Mathematics in their achievements. . . . The modern man, therefore, should have at least some conception of the means by which these results, so vital to him, are obtained." We have to give to our boys a training in logical accuracy, a demonstration of the beauty and interest of mathematical ideas and methods in themselves, and in the words which I have quoted, "some appreciation of those problems of the world, the solution of which depends on Mathematics and Science." "The majority of boys will not make any extensive or profound use of Mathematics in after life . . . but they can be taught to realise the tremendous potentialities of the study whose elements they are commencing. . . . A public must be created able to realise what Science and Mathematics are doing for the world and to form some general conception of the means employed."

It can hardly be maintained at the present time that this ideal has been attained. I can give you one illustration.

Some two years ago a suggestion was made that the Elements of Quaternions should be taught to the highest mathematical sets of our schools: an able mathematician remarked somewhat regretfully in a discussion of this suggestion that he would have to do a good deal of reading before *he* could take his share of that work. "I never knew much about Quaternions, and I know less now." A well-known man to whom he was speaking, a distinguished classical scholar, expressed his astonishment: "You don't mean to say that you've forgotten your Euclid"—and in reply to a somewhat hesitating question as to the use of Quaternions by Euclid, he continued triumphantly: "Evidently I remember these things better than you do, although you're teaching them every day. You really don't mean to say that you have forgotten Euclid's definitions: 'A four-sided figure with its opposite sides parallel is called a parallelogram and every other four-sided figure is called a quaternion.'"

I have commenced my paper with that story because it illustrates quite successfully the attitude of the majority of so-called educated people. If I were to declare that Plato wrote the *Aeneid* or that Lucretius was an Athenian statesman, immediately and quite rightly my ignorance would make me the laughing-stock of all who knew me. But just because the average educated man is entirely ignorant of even the most elementary principles of Mathematics, colossal ignorance is allowed to pass almost unnoticed, even when that ignorance extends to the whole of those methods which have made possible so much of modern science and industry. Improvement in this respect is retarded by the increasing tendency to specialisation. Thus the boy of rather above the average ability in subjects literary in most of our schools begins to specialise at the age of sixteen (or even earlier). He often knows that Mathematics will not be a principal subject in any examination for which he is going to enter: often he is firmly convinced that Mathematics simply consists of a series of tricks for solving puzzles in which he is not particularly interested, and he contrasts in his mind the joy of reading a classical author or a book on mediaeval or modern history with the boredom

which comes from the solving of a series of puzzles, more or less of the same type, entirely without artistic beauty, entirely unrelated to life. Even if he has a certain facility in solving these problems, even if he realises the achievements of Mathematics in the modern world, still the romance of the subject, the aesthetic value of many mathematical methods, the possibility of his envisaging the subject as a whole, are entirely hidden from him. There is a further point which must be remembered, viz., that such a boy will tend to be in comparatively low sets for Mathematics, and that inevitably the best teachers are put to teach the boys who are more promising as mathematicians. The result is that the boys who are most ready to appreciate the historical side of Mathematics, who often have a wider outlook on life than the mere mathematician, are the very boys who have least opportunity of realising the relation between Mathematics and life, or the place of Mathematics in the scheme of culture as a whole.

I have been thinking primarily of classical specialists, but the same is equally true of those who specialise in Modern Languages or in English subjects.

I would suggest that at Upper Fifth Form standard (whether it be on a classical or modern language or history side), a boy whose interests are not mathematical and who shows no real proficiency in Mathematics should be withdrawn from the ordinary sets and placed in a special set to be taught without any reference to the ordinary scheme of the school. This would, of course, tend to raise the standard of the science side sets. At present, because classical scholarship candidates cannot spare the time for the full mathematical course, they are deprived at most schools of all mathematical teaching whatever, and we are turning out a number of our ablest boys unable to explain intelligibly what Mathematics is about—unable for instance to realise that a quaternion is not a quadrilateral. Nor do I believe from the somewhat limited experience I have had that the abler History or Modern Language boys whom I have known have derived much benefit from their attendance at the ordinary mathematical classes of the school. Thus in the two *bottom* sets of the A-block at one school there are fifteen boys of Sixth Form rank, while ten boys have ceased to do Mathematics at all, out of a total of fifty-five in the Classical, Modern and History Sixth Forms. Those twenty-five could quite well form the special set, and the ordinary sets would be relieved by their withdrawal.

Now we must remember that such boys will probably be baffled entirely by any amount of algebraic or trigonometrical manipulation. On the other hand, curiously enough, Geometry taught on the compromise lines of most of our modern text-books seems to appeal to such—the idea of a connected reasoning from clearly defined premises to a conclusion such as Pythagoras' theorem, which is not obviously true, certainly makes an appeal to the boy of average intelligence, and whatever our course for this special set does omit, it should *not* omit examples of clear reasoning. A further difficulty is caused, that boys would come to such a set at different stages of their mathematical career, and therefore the curriculum of the special set should, so far as is possible, avoid overlapping any part of the ordinary course. The one thing to be avoided with such a set as I have described is the repetition of work which perhaps half the boys have done already.

I would suggest as subjects for the consideration of such a set:

1. The idea of variation and functionality, including the idea of gradient, of differentiation and integration.

2. The idea of number.

3. The idea of periodicity. I need not enlarge upon the educational value of the ideas of variation and functionality. Prof. Whitehead has advocated a great extension of the graphical study of statistics: "There are to our hands statistics of trade, both external and internal, statistics of railway traffic, statistics of crime, of income-tax returns, of national expenditure, of

weather and so on. The reduction of these to graphs, the careful study of the peculiarity of these graphs, the search for correlation among them, and the study of public events which corresponded in time to peculiarities of graphical form, would teach more Mathematics and more knowledge of modern social forces than all our present methods put together." So most modern text-books of Algebra contain quite good chapters on variation, but I fear that few boys of the type I am primarily considering have a clear idea of the meaning of the statement that the attraction of the sun for a planet varies inversely as the square of the distance between them. However, I have found that the more intelligent boys are always interested in the subject of variation when treated graphically, especially as it gives the master untold opportunity to wander from the point into realms of Astronomy, Electricity, or what not. Further, this study requires very little power of manipulation of algebraical expressions. The use of logarithms sometimes can shorten numerical calculations, but in this case I see no real objection to the boys using the tables without a very clear idea of the reasons for the method.

From functionality in general we naturally come to consider the gradient. Here it seems to me that since the manipulation of algebraic forms is not of primary importance, time could be well spent in expounding clearly the meaning of a limit. These are useful helps in the use of the method of limits in elementary geometry of the circle, and especially in the interesting proof that $\frac{\sin\theta}{\theta}\to 1$ as $\theta\to 0$. When the idea of the limit of a function had been cleared up, the example of the gradient of a function might be considered and a first course in Calculus covered, as in Godfrey and Siddons' *Algebra* or Mercer's *Calculus for Beginners*, the object being always to show how problems of maxima and minima, of velocity and acceleration, of work and energy and momentum are solved by the Calculus. The use of Simpson's rule for the determination of areas always seems to interest boys.

Again the history of the Calculus, *e.g.* the Newton-Leibnitz controversy, illustrates the way in which the thoughts of men are gradually widened, and especially the way in which, in a note-worthy fashion, thought seems to stagnate for long intervals of time. No progress is made for centuries until the necessary new weapon—such as the Differential Calculus proved to be—has been discovered. Then progress is made by leaps and bounds until the potentialities of that particular weapon have been exhausted—and then stagnation, for a time.

I wish to lay stress on a discussion of the idea of number. The boy I have in mind remembers quite well the insistence with which he was taught that in multiplication two minuses made a plus, and if he has advanced very far in the Algebra books he may have discovered the "existence" of the mysterious square root of -1, and had his objections to the existence of such a number over-ridden, or at any rate never properly discussed. "I never saw why . . ." is the commonest criticism of the mathematical master urged by such boys. I think a course could be worked out on the lines suggested by Prof. Nunn in his *Teaching of Algebra and Exercises in Algebra*, part 2—discussing successively numbers, directed numbers, fractions, irrationals, complex numbers. The idea of an irrational such as $\sqrt{2}$ can be introduced, by some such method as that given by Hardy in his *Pure Mathematics*—by considering the two sets of numbers, 1, 1·4, 1·41, 1·414, 1·4142 ... and 2, 1·5, 1·42, 1·415, 1·4143 ..., or if desired the formal theory of the Dedekind section could be discussed. Then the complex number $a+ib$ must be regarded as a pair of numbers, corresponding to the point (a, b) of the plane. From the complex number a valuable idea of the generality of mathematical work can be derived, such as that expressed by Prof. Whitehead when he likened the mathematicians' use of symbols to Tweedledum's use of words: "I pay them extra and make them mean what I like." I want my well-educated boy to be in

a position to appreciate the dialogue in Prof. Eddington's book on *Space, Time and Gravitation*, in which the mathematician accepts the statement that he not only does not mind whether his propositions are true, but does not even care to know what he is talking about.

Again the change in connotation of the word "number" in the last three hundred years seems to me to provide a most valuable illustration of the ways in which men think—the extension of the child's concept of whole numbers to fractions, of non-directed numbers to directed numbers, and so on. Clifford's *Common Sense of the Exact Sciences* gives a good idea of how to approach Demoivre's theorem, and in this way the extension of the number concept to include quaternions might be shown. Might I repeat that with many boys one's aim should not be to attain great facility in algebraical work, but that, for example, in connection with quaternions, a so-called educated man should have some idea of what a quaternion is and how its use resulted in a simplification of mathematical reasoning and a consequent progress.

On the subject of periodicity I only propose to dwell for a short time. To quote Prof. Nunn: "Periodicity in physical phenomena is among the most familiar of our experiences—'seed time and harvest, cold and heat, night and day'—being only the most conspicuous instances." The way in which a formula connecting variables, one of which is a periodic function of the other, can be built up is of considerable interest and no great difficulty. Probably the easiest introduction is by means of a reference to the simple pendulum. From this it is of interest to learn that the mean temperature during each month at Greenwich for many years has agreed tolerably accurately with the formula

$$T = 50{\cdot}6 + 12{\cdot}8 \sin 6{\cdot}9W,$$

T being the temperature in degrees Fahrenheit and W the number of weeks from April 15th, 1910. This graph and the statistics on which it is based would provide a good introduction to the type of equation representing wave-motion.

From this the problem of harmonic analysis could be attempted—in a few simple cases: enough could be done to show how the mathematician is able to predict the time and height of high tides. Again, I should like to emphasise the fact that I am not so much concerned with the actual manipulation of algebraic and trigonometrical expressions as with showing boys why Mathematics has played such an extensive part in modern science and the kind of method which has been used.

There is one part of the subject which seems to me to be of great value—a course in Geometry—perhaps a hasty course of Euclidean Geometry, including a clear statement of the axioms and postulates, not merely those explicitly made by Euclid, but those whose discussion has resulted in the formulation of the non-Euclidean geometries of more recent days. Would it be possible to include a sketch of these geometries? May I quote a remark of Prof. Carson—"I am bound to state my conviction that every student of whatever subject who proceeds to a University education worthy of the name, should gain some ideas of the nature of non-Euclidean geometry. Their appreciation throws a light on the space-concept in particular and our so-called knowledge in general which can be gained in no other way. It may be fairly said that there are few portions of mathematical knowledge which have more educational value."

That a course in the principles of Statics and Dynamics will be of value few would, I think, deny. The only difficulty will be that possibly several of the class would have done a certain amount already and some repetition would be necessary. At the same time I know of no part of the mathematical syllabus which so obviously bears on practical life and of which the history of the development is so full of interest.

Few controversies are of such interest as the eighteenth century discussions as to how the so-called "force of a body in motion" was to be measured,

leading to the resolving of the problem by D'Alembert by the distinction between kinetic energy and momentum.

Some years ago, Mr. Newbolt, then a mathematical master at Tonbridge, and a member of our association, had a class consisting of the Classical Upper Sixth Form, two hours a week for a year, and in a paper in the 1912 Board of Education Report on the Teaching of Mathematics in the United Kingdom he described his experience and included the examination papers which were set to the boys at the end of the year. For various reasons, none of which I think were relevant to our discussion, the experiment was not continued, and so far as I know, no further attempt of that type has there been made. But Mr. Newbolt seems to have been convinced of the value of such a plan—"The results of the year's work," he says, "as well as the interest shown throughout by the boys themselves amply justified the experiment." S. H. CLARKE.

Mr. C. V. Durell and Mr. W. P. Milne took part in the discussion on this paper.

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71. Henry Clarke.

Practical Perspective, Lond., 1776, 8vo, vol. i.; ? all dedicated to Charles White, Esq., F.R.S., of Manchester.

The Rationale of Circulating Numbers, London, 1777, 8vo. Dedicated to Thomas Butterworth Bayley, Esq., of Hope, F.R.S. Here he advertises an Essay shortly to be published on The Usefulness of Mathematical Knowledge.

A Dissertation on the Summation of Infinite Converging Series, London, 1779, 4to. Dedicated to Hutton. This was translated from Latin of A. M. Lorgna.

A Supplement to the last named, London, 1782, 4to.

Some of his papers are said to be in the T. T. Wilkinson bequest of MSS. to the Chetham Library.

In the 1816 Dictionary of Living Authors there is one of his name LL.D., Professor of Astronomy and Experimental Philosophy at the R. M. College, Marlow.

The Seaman's Desiderata, 4to, 1800.

A New . . . Operation for clearing the apparent Distance of the Moon, etc., 4to, 1800.

Animadversions on Dr. Dickson's Translation of Carnot's Reflections on the Theory of the Infinitesimal Calculus, 8vo, 1801.

Printed for and sold by Ogilvy and Speare, Middle-Row, Holborn, 1794;

"In this list of books by Henry Clarke, Praelector on Philosophy in the College of Manchester, the first above, illustrated with 33 copper-plates and moveable Schemes, 8vo, 5s., boards. The second, 8vo, 4s., boards; the Dissert., 4to, 10s. 6d., bd.; and the Supplement, 4to, 2s. 6d., sewed.

Add to above: Additional Remarks on Converging Series, occasioned by Mr. Landen's Appendix to his Observations on the same subject, 4to, 1s. 6d., sewed."

His LL.D., 1802, was from the University of Edinburgh, when appointed at Marlow.

He wrote a Treatise on Shorthand and an Introduction to Geography.

72. Mortimer Collins had a theory that mathematics and poetry were allied, and commenced a work on "Rhythmic Algebra," but abandoned it because he was advised that the public would not be sufficiently interested in the subject to make the work pay, and he could not afford to work without payment.—*M. Collins' Letters and Friendships*, by Frances Collins, 1877, ii. p. 219.

THE EARLY HISTORY OF THE ASSOCIATION,

OR

THE PASSING OF EUCLID FROM OUR SCHOOLS AND UNIVERSITIES, AND HOW IT CAME ABOUT. A STORY OF FIFTY YEARS AGO.

Notes of the Presidential Address to the Mathematical Association on 4th January, 1921, *by* CANON JAMES M. WILSON, D.D.

I MUST in the first place thank you most warmly for the great and very unexpected honour you have conferred upon me by inviting me to be your President and to relate some incidents connected with the birth of this Association fifty years ago. I do not think that any honour I ever received has given me more pleasure.

May I, as by courtesy and custom bound, begin by saying a word or two about our distinguished retiring President, Professor E. T. Whittaker, whom I now for the first time have the honour of meeting?

You have lately, in the *Mathematical Gazette*, been preserving anecdotes about famous mathematicians of the past, such as Maclaurin, De Morgan, Babbage, Sir W. R. Hamilton. I shall therefore be quite in order if I try to preserve an anecdote or two about Whittaker.

When I was Vicar of Rochdale and Archdeacon of Manchester (1890-1905) Whittaker was a boy at the Manchester Grammar School. Some of his school-fellows from Rochdale and its neighbourhood often went up with me by the same early train to Manchester, and we became acquainted with one another. They used to boast of having an extraordinary genius in their school. "He could solve any problem. He knew more than any master. Would I try him?" I said he would regard it as an impertinence. "Not at all," they said; "he would like nothing better; but it must be a stiff question." So I consented; and finding that he was about sixteen, I asked them to get him to prove that the sums of every third coefficient in a binomial expansion, whether beginning with the first or second or third, were either equal or differed by 1. To my surprise the boys brought me his solution in a day or two; and then I learnt that his name was Whittaker.

Of course the boys were triumphant. They defied me to puzzle him. Now an Archdeacon who had been a senior wrangler could not tamely submit to be beaten by a mere school-boy. Besides my own credit the reputation of the clergy and of Cambridge itself was at stake. So I wickedly gave him a very innocent looking, but difficult, problem—"To shew that every prime number of the form $4n+1$ is the sum of two squares." The boys shouted that he would soon solve that. They little knew. I enquired from time to time for the solution. "He has not done it yet," was the answer. Weeks passed. Then they told me that he had given it to the masters, and it had stumped them. So my reputation was saved.

I remembered it as occurring in the second or third of H. J. S. Smith's four reports to the British Association on the Theory of Numbers. I know of no property of numbers more curious.

May I save for posterity another story of Whittaker? In 1902 he had become a fellow of Trinity College, Cambridge; and I heard from my friend H. H. Turner, that Whittaker had found a solution of the famous differential equation arrived at by Laplace in 1787 as the key to a problem about Saturn's rings, and shown subsequently to be the key to many other physical problems; but the solution of which had defeated Laplace and all subsequent mathematicians for 120 years. I gave an account of this equation in the *Manchester Guardian* of 2nd September, 1902, under the title "A mathematical discovery by a Manchester man."

With his later distinctions and contributions to science you are better

acquainted than I am; and I doubt not that we may apply Browning's words to Whittaker's work—"The best is yet to come."

I now proceed to the special subject on which you have invited me to address you—the early history of this Association.

I feel sure that you will agree with me that the story I am to tell ought to begin, as biographies do, before the birth of the child, by tracing its immediate ancestry. And this part of the story, which is necessarily the least known to the present members of the Association, and the least easy to recover, will form the main subject and perhaps the most interesting part of my Address. It will show how necessary the Association was, and what good work it has done; but I think it may also surprise you to learn how much progress had been made in the Reform of Geometrical Teaching in Great Britain before this Association was in existence.

The earliest thorough criticism of Euclid as a text-book of Geometry that I can recall came out in 1849. In the *Companion to the British Almanack* for that year Augustus De Morgan—a great name among English mathematicians—drew attention to Euclid's imperfections in a subtle and original paper. But it attracted little or no attention among teachers of mathematics. Euclid's reign in England remained as unshaken and unquestioned as that of Queen Victoria.

In France, from Napoleonic times until about 1840, the text-book of Legendre was in ordinary use. It was regarded as a happy modification of Euclid. But about that time the professors of mathematics in France began to depart more and more widely from their model; and by 1860 it had become, it is stated, the universal custom in the Lycées and Universities for each professor to prepare his own course of oral instruction in Geometry. The students took notes of his lectures, and in fact made their own text-books. These courses closely resembled one another, because the public examiners issued detailed programmes of the subjects required for each examination. The nature of the Geometrical Teaching in France was thus decided by the Syllabus of the Government Examiners in Geometry.

The attention of English teachers had been little, if at all, directed to French methods until, I think, the middle or late sixties, when Napoleon III sent Mm. Démogeot and Montucci to England to report on English Education. On the Mathematical teaching in our schools they reported very unfavourably. One of their chapters begins with the following remarkable words, "Le trait distinctif de l'enseignement des mathématiques en Angleterre c'est qu'on y fait appel plutôt à la mémoire qu'à l'intelligence de l'élève." They remark later on, "On y trouve (in Euclid) sans doute une logique de fer, qui n'admet point de réplique; mais aussi arrive-t-on aux résultats les plus évidents par un verbiage absolument en désaccord avec nos élégantes habitudes d'une concision non moins vigoureuse que la prolixité d'Euclide."

I have no recollection that this very interesting volume attracted much attention in England. It was specially interesting to me because M. Démogeot came to Rugby, and in particular spent some time in my boarding house, paying a quite unexpected visit to the boys' studies one evening. I had met him at dinner at the School house; and he expressed surprise that I could leave my boarding house without any supervision. So I took him to see what every one was doing under no supervision. He was astonished that the boys so thoroughly deserved the trust placed in them.

As far as I know, the first step taken in England in the direction of the French practice was the issuing of programmes of their mathematical examinations by the professors of mathematics in the Universities of Oxford and London. Under these programmes Examinees were not limited to Euclid's proof or style. Cambridge, however, did not as yet move. Whether this change in Oxford and London preceded or followed the Report of the French Commission I do not know.

The important and decisive event took place in 1867. The Report of the "Schools Inquiry Commission" was published: and this Report made plain to all a fact, in which schoolmasters had hitherto acquiesced as a decree of fate, that boys might have worked for years at Euclid, and even know Euclid perfectly, and yet know next to nothing of the spirit or method or the results of Geometry. Time was in fact wasted over Euclid: and time was of importance. A better method was wanted.

Dr. Temple, afterwards Archbishop of Canterbury, was at that time Head Master of Rugby School, and a very leading member of the Commission. He was a very able mathematician. I was then the senior mathematical master of the school, and Dr. Temple asked me to prepare and publish a text-book of Geometry, more suitable than Euclid for the Public and Grammar and Secondary Schools which the Commission had in contemplation. I set to work at once, making use of the best text-books I could get from France, notably those of Amiot, Briot, and that of Rouché and de Comberousse. I also used a popular German text-book by Meyer, and Peirce's text-book from the United States; and several others whose names I forget. The first edition of my Geometry was published in April, 1868, and did not go farther than Euclid, Books I. and II. This was, I believe, the first challenge to Euclid in England. In my preface I gave some account of its origin and purpose, and acknowledged obligations to Dr. Temple, to the Rev. C. E. Moberly (a classical colleague whom I described as having "the spirit without the prejudices of the geometrician"), and Professor Hirst of the University of London. We began to use it at once in Rugby School. It was studied by the boys with extreme care, because I offered sixpence for every mistake and misprint, however trifling.

The distinctive features of the work were stated in the Preface to be as follows: "The classification of Theorems according to their subjects; the separation of Theorems and Problems; the use of hypothetical constructions; the adoption of independent proofs where they are possible and simple; the introduction of the terms locus, projection, etc., the importance given to the notion of direction as the property of a straight line; the intermixing of exercises classified according to the methods adopted for their solution; the diminution of the number of theorems; the compression of proofs, especially in the latter part of the book; the tacit, instead of explicit, reference to axioms; and the treatment of parallels."

The sections of the first book were entitled "Angles at a Point; Triangles, Parallels and Parallelograms, Problems, Equality of Areas, and Problems." Each section was followed by numerous associated and appropriate exercises. At the same time my colleague Mr. Kitchener and I wrote and published a separate little text-book of *Experimental Geometry and the use of Simple Instruments, as an Introduction to Theoretical Geometry.*

This *Geometry* of mine attracted some attention, and had a fairly rapid sale, specially in girls' schools, during 1868 and 1869. It was severely handled by De Morgan, in, I think, the *Athenaeum.* I had a good deal of correspondence about it, and spoke at various gatherings of teachers; and at a meeting of the London Mathematical Society. Of my address there I have no notes; but I was well "heckled." One of these addresses was of considerable importance, and I still happen to possess it. It was delivered in the Royal High School of Edinburgh on January 22nd, 1870, after a letter of welcome from a really extraordinary list of distinguished citizens of Edinburgh and other Scotch cities, which I also still possess. This letter is signed by the Lord Provost of Edinburgh, by the Convener of the Education Committee of the Town Council; by the two Members of Parliament for the city; by the member for the Universities of Edinburgh and St. Andrews, and by the member for those of Glasgow and Aberdeen, and a long list of Professors in many faculties and Headmasters and others.

I mention those signatories, because nothing else can so convincingly show how much interest the question had already in 1869 aroused in literary and scientific circles in Scotland, and how much the reform had advanced. For the same reason it is, I think, worth while to quote the opening paragraph of the letter sent to me. It begins thus:

"We, the undersigned, duly appreciate the great advantages which have already arisen, and may still be anticipated, from recent reform in the process of teaching Geometry. In France the syllogistic system of Euclid has long been superseded by more direct and practical methods of proof, and these methods have been introduced, with complete success, into many of our English and Scottish Schools. We believe, however, that professional teachers and the general public need further information regarding this scholastic experiment; and we are gratified to hear that you have agreed to deliver in the Hall of the High School of Edinburgh a lecture explaining and vindicating the reformed method of Geometrical Teaching; inasmuch as we are convinced that benefit will thence accrue alike to Education and to Science."

It was a large and very remarkable gathering, Professor Kelland, the senior wrangler of 1834, being in the chair. He was, however, wholly and openly opposed to the change: and I see that in thanking him in my opening sentence for consenting to take the chair, I said that "should Euclid finally pass away from our schools it will be a melancholy satisfaction to his friends to know that he had the ablest advocate in Edinburgh retained for his defence."

The address was printed by Billington of Rugby, and sent out fairly widely. I quote from the final paragraph which briefly summarises the address:

"I showed," I said, "that in teaching Geometry some familiarity with facts should precede the study of the deductive science; that this science should be kept clear of metaphysical questions, and be distinguished above all others for its naturalness, orderliness, intelligibility and accuracy; and these are qualities not inconsistent with constant practical applications. I then endeavoured to show that Euclid's elements fail in method, consistency, and in elegance; that the book exercises the memory rather than the reason; that it fails in suggestiveness and in utility; that it is disfigured by errors of detail and of principle; and that it is as inadequate and unsuitable a text-book of Geometrical Science as might be expected from a writer of twenty centuries ago, even though that writer had the masterly and consummate ability that Euclid had."

I concluded by reminding them that a Committee of the most distinguished mathematicians in the British Isles had been recently appointed to consider the possibility of improving the methods of instruction in Elementary Geometry. The names of the members of that Committee were Sylvester and Cayley, H. J. S. Smith and Price, Kelland and Fuller, Salmon and Townsend, Hirst, Spottiswoode and Clifford. Was there ever such a cluster of stars on any Committee? Whether that Committee ever met or reported I do not know. It certainly did not produce a syllabus. I think I should not have forgotten had it ever reported.

But the interest in the question, as decisively shown by the Edinburgh gathering, was increasing; and it had become practically certain that a revolution in geometrical teaching was pending. During the year 1870 correspondence took place among mathematicians and mathematical masters with the aim of securing sufficient uniformity of treatment to make possible the conducting of Examinations. That was the real difficulty. Some of us, including myself, had hoped that either the British Association, or an association of University Professors would draw up for us an authoritative Syllabus to which text-books might be written. But those august bodies made no sign. We therefore resolved to form an Association for ourselves, since the need was urgent, in order to enquire into the matter carefully, and subject our

opinions to the criticism of one another. And now at last we have come to the birth of the child.

For the actual birth I have to depend on the memory and memoranda of another. Some of you will remember the amusing scene at our Annual Meeting held ten years ago, on January 11th, 1911. I had brought before the Association an account of a thirteenth or fourteenth century MS. in the Worcester Cathedral Library. It contained a copy of part of the translation of Euclid from an Arabic original into Latin, made in A.D. 1120 by Adelhard, a monk of Bath. Some of you will also remember the extremely interesting discussion in which Professors Hobson, Harding and Turner took part, reported in the *Gazette* for March, 1911. On that occasion Professor Turner and I both expressed our belief that I was probably the only survivor of the original members of this Association, but the Rev. W. Done Bushell rose at the close of my remarks and corrected our impression. He also was one of that original council which met, though I had forgotten the fact, in my own house at Rugby on January 17th, 1871, and he produced the actual agenda of that meeting in my own handwriting. He compared me then, you may remember, to Moses, " whose eye was not dimmed, nor his natural force abated " after forty years' wandering in the wilderness. Mr. Bushell, however, has passed away. I do not know to whom you can compare me now, except Methuselah.

That is, I fear, a slight digression, but not irrelevant to our history.

To return to our Association. Our early council discussed at length whether we should jointly publish a text-book, or only prepare a syllabus. I advocated the former course, and placed my book entirely at their disposal. But the Committee, I now think wisely, chose the latter. We appointed a Sub-Committee, of which the very able London University Professor, Hirst, was the Chairman, to draw up the syllabus. Of that Sub-Committee I was not an active member. It met in London, and I could not regularly attend. They took my syllabus as their basis, and improved upon it. The Committee was very slow in its work ; by the end of 1873 it had only finished the syllabus up to, but not including, Proportion ; but what it did has stood the test of time. In 1872 I published a *Solid Geometry and Conic Sections*. In 1873 I published the third and larger edition of my *Geometry*, including the circle, following the syllabus of our Association ; and here I mention with pleasure that my fellow worker in Book III. was Mr. J. F. Moulton, senior wrangler of 1868, now Lord Moulton, G.B.E., K.C.B., Lord Justice of Appeal. We were also fellow-conspirators in exploding the mathematical and physical science of the first book of Herbert Spencer's *First Principles*.

The rest of the history of the work of the Association has passed from my memory, but it can doubtless be recovered from its minutes. I cannot even recall the names of any members of our original committee, except W. D. Bushell and R. B. Hayward from Harrow, Spottiswoode, Hirst, Levett, and I think MacCarthy.

It soon enlarged its scope from being an Association for the Improvement of Geometrical Teaching alone, to include all Elementary Mathematics. For some years the progress was slower than was expected. But reason prevailed ; and at the present moment, thanks to this Association, the teaching in all British schools in Arithmetic, Algebra and Geometry is very much better than it was fifty years ago. Every one of the reforms we then pressed for has been adopted in the newer text-books. Such a text-book as that of Godfrey and Siddons, written to such a Syllabus as that published in the *Regulations of the Oxford and Cambridge Examinations Board for the year* 1921, fulfils all our ideals of fifty years ago, or nearly so, and adds to them in some respects.

I say " *or nearly so*," because we then attached much educational importance to the giving some clear notion of the nature and the use of Geometry : taking Comte's definition of Mathematics—" as the *indirect* measurement of magnitudes, and processes subsidiary thereto—" and applying it in the first

instance to Geometry. Every theorem may be shown to be a means of indirectly measuring some magnitude. That the angles at the base of an isosceles triangle are equal shows that if you can measure one, which is accessible to you, you have indirectly measured the other. Other theorems obviously illustrate a range-finder, and astronomical measurements. No pages in my third edition were educationally more useful than pp. 4, 5 on this topic.

We also attached importance to giving boys some notion of the wide application of mathematics—to everything included in the word *magnitude*. Not only length and area and volume, but everything measurable in terms of a unit—velocity, force, temperature, heat, light, electric power, tidal energy, aeroplane dynamics, all form the field for mathematics. From the problems of sidereal astronomy to those of the atom, from the infinitely great to the infinitely small, all knowledge is got by indirect measurement, that is by Mathematics. I think a few paragraphs on this subject, and on problems still unsolved, will interest boys and girls, widen their range of knowledge, and what is of no less importance, train their imagination. All this appeals to boys and girls whose chief interests are non-mathematical. I would venture also to suggest to the Association that the Psychology of the learner may prove a fruitful subject for study by the teacher. We should all admit that the order of teaching mathematics and science should have a close relation to the development of the learner's curiosity and interest in the subject taught. It is a guide to his capacity. And it is highly probable, I think, that this development in the learner's mind will be found to be generally parallel to the historic order of study and discovery—the development, that is to say, in the mind of the "colossal man," the human race. Pardon these suggestions from an old mathematical master.

Finally, let me express my regret that the address has been unavoidably egotistical, and my thanks to you for so patiently listening to my long story.

JAMES M. WILSON.

P.S.—The Editor of *Nature*, in the issue for Jan. 13, 1921, has given a valuable historical supplement to my recollections as given above, showing the important part played by that Journal in pressing Geometrical Reform. From his note we also learn that my address to the London Mathematical Society, spoken of above, was printed in the *Educational Times* for September, 1868; and that Mr. Tucker, of University College School, and Mr. Wormell were among the prime movers in the Reform. We learn also that I have understated the part I played in preparing the syllabus; that it was my suggestion in 1871; and that I worked on it till 1874, and "took the foremost place in the actual work of getting out the syllabus." So much for my memory. I thought I had disappeared into the background by that time.

P.P.S.—Since this address was in print Lord Moulton has passed away.

J. M. W.

Here it is convenient to append a few recollections supplied by two surviving members of the earlier days:

FURTHER REMINISCENCES.

MR. A. A. BOURNE.

I HAD the privilege of being one of the early members of the Association for the Improvement of Geometrical Teaching. I believe that my joining was due partly to my brother, C. W. Bourne, and partly to J. M. Wilson (now Canon of Worcester), who was my housemaster and mathematical master at Rugby, and to whom I owe very much. The latter was certainly one of the most active members in the early days, when we met at University College in Gower Street under Dr. Hirst as President. After the lapse of nearly half a century one's memory of details is apt to be meagre, and unfortunately it

is sometimes just the unimportant parts which most readily reproduce themselves. I know that we had very pleasant meetings. Masters and lecturers, comparing views and experiences, could look forward hopefully to a greater freedom in Geometry. We were not wildly revolutionary; but we were warned, I believe, to give heed to the example of Italy, where the abolition of Euclid had led to such pernicious confusion in the teaching of various geometrical systems that the re-establishment of Euclid was actually decreed by authority.

In the meeting of 1872 came the examination of programmes (for text-books) drawn up by various members in accordance with a resolution of the 1871 meeting.

Resolutions were passed approving certain details or principles to be adopted in any syllabus. The matter which gave most chance of a fight was a list of proposals drawn up by W. H. Laverty. I do not think these met with as kind a reception as they deserved. The most important of them were the following: "That the separation which has hitherto been maintained between the methods of Algebra and Geometry is artificial." "That it is useless to draw a distinction between commensurable and incommensurable quantities, seeing that by use of Infinitesimals incommensurables may be brought under the same methods of proof as commensurables."

Then there was the question whether a text-book or only a syllabus should be produced by the Association. A few members thought that a book might be written: it was felt, however, by the majority, and strongly expressed by the President, that while the preparing of a syllabus was a suitable and necessary task for the Association, "no really valuable text-book would ever be produced by a Committee or an Association."

In 1873 came the chief discussion of the proposed syllabus. There was pretty general agreement that

(1) Practical should begin before Theoretical,
(2) Problems should be separated from Theorems,
(3) Numerical examples should be freely used,
(4) Loci should be treated.

Oxford (in the Local Examinations) had already decided to allow "the portion of Geometry treated in Euclid" as an alternative; but Cambridge had made no relaxation from its rigidity. Dr. Hirst maintained that if we improved geometrical teaching, the examiners would find some means of testing the knowledge of candidates so trained. (As our chance of improving geometrical teaching seemed greatly dependent on recognition by examiners, was not this something of a "vicious circle"?) The Committee were asked to consider and report to the next meeting the best way of bringing the work of the Association to the notice of Examining Boards.

Later in the year the A.I.G.T. was greatly encouraged by the compliment of recognition and praise of its work by the British Association, a recognition which was repeated in 1876.

In 1874 there came up the subject of Proportion. Of course there was much argument on the best way of treating the subject. No one could pretend that it was a simple matter to interest boys in the subtlety of the fifth definition of Euclid, Book V. How was one to deal with this obstacle? Should one get round it by postponing all reference to Incommensurables, or climb over it by means of Infinitesimals. On J. M. Wilson's proposal it was decided that all the five proposed schemes of Proportion should be submitted to all members with a request for an opinion on the best method to be adopted.

In this year was held the first examination for certificates under the newly formed Oxford and Cambridge Schools Examination Board. Their programme allowed candidates to offer as Geometry "the parts treated of in *Euclid*, I., II., III., VI. 1-19, and A"; but the papers set showed no particular departure from *Euclid*. About the same time the Cambridge Previous

Examination began to admit proofs "which follow Euclid's order," and to insert among the questions some easy exercises.

By the end of 1877 the Association had gained a good deal of recognition for its published syllabus of Plane Geometry; and the next meeting in January, 1878, appointed a sub-committee to tackle the subject of Solid Geometry.

At the same meeting much discussion was provoked by an important motion of my brother's, "That in the opinion of this Association it is both reasonable and expedient that candidates at all Examinations in Elementary Geometry should be required to give evidence of such ability as is necessary for the solving of easy Geometrical Exercises; and that the Secretaries of the Association be instructed to send a copy of the Resolution to the leading Examining Bodies of the country."

After considerable discussion in which it was pointed out that by this resolution we should be dictating to Examiners, it was decided to pass it in a modified form, viz., "That it is highly important that further encouragement should be given to candidates at all examinations to give evidence of such ability as is necessary for the solving of easy Geometrical Exercises; and etc."

After this and the appointment of a committee for the consideration of Higher Plane Geometry, and one for Geometrical Conics, ended the last of the important meetings which I was able to attend for some time. I think there was an intermission of meetings. At any rate I will end here my reminiscences.

Many had a share in the success of the A.I.G.T. up to this point; but with all apologies to some whom I may have forgotten I venture to mention the joint secretaries, E. F. MacCarthy and R. Levett as well as such protagonists as J. M. Wilson, J. F. Moulton, R. B. Hayward, Alexander Ellis (a venerable and industrious critic), and C. W. Merrifield, who with others contributed to a notable success.

Mr. F. S. Marshall.

In 1870 the mathematical teaching at Harrow, on the classical side, was entirely without organisation. The forms grouped in four blocks were handed over for certain hours to the mathematical masters. They divided each block amongst them according to fancy or some chance acquaintanceship with the boys. The senior mathematical master, the Rev. R. Middlemist, was a relic of a previous age. He taught, if teaching it could be called, on lines that had become stereotyped twenty or thirty years before. His conception of the schoolmaster was that of a somewhat savage disciplinarian, and his watchfulness did not prevent the same exercise being shown up by the same boy for a whole term or even a year!

Robert Baldwin Hayward, Fellow of St. John's College, Cambridge, was in most ways a contrast to this. He earned the Fellowship of the Royal Society in 1876 for his distinguished work in Mathematics.

He was one of the very few schoolmasters who after some years of work at school have been invited to act as examiners for the Mathematical Tripos at Cambridge. An elegant and accomplished mathematician, he might have done valuable original work had he not been preoccupied by the daily grind of elementary teaching. He was an active and important member of the Committee of the A.I.G.T. from its start, and impressed on his colleagues the need of a freer handling than Euclid's of Geometry. As a teacher of a low class he was out of his element; as a teacher of individual pupils or of a few pupils of ability he was excellent. As a pupil of De Morgan he set a high value on the intelligent teaching of Arithmetic, and in his, all too rare, discussions of their work impressed his ideals upon his juniors, A. C. Tosswill and F. E. Marshall, newly come from the more highly organised school at Rugby. In 1871 he proposed to them that the boys handed over to them in blocks

should be sorted in divisions according to their attainments or ability, and progress began. The publications of the A.I.G.T. were studied, and Euclid discarded for a time in the lower forms for a text-book embodying its ideas. This and some experiments in geometrical drawing permanently affected the teaching, though Euclid continued to be used as the text-book of Geometry in the higher and ultimately in all the forms.

The Rev. W. Done Bushell, Fellow of St. John's College, Cambridge, had been a master for two terms at Clifton under Dr. Percival before he came to Harrow in 1866. When the modern side was started in 1869, Mr. Bushell took charge of the Mathematics taught therein, and proceeded to organise them elaborately, even classifying the boys in different groups for geometrical and analytical work. His geometrical teaching was designed to insist on an intelligent and not a slavish study of Euclid with frequent practical applications. Mr. Bushell had no mathematical colleague who devoted his whole time to the modern side. Consequently the Time Table was a veritable Chinese puzzle, for the hours had to be fitted in when sufficient members of the mathematical staff were at liberty from the teaching of the classical side.

As the modern side grew from the size which demanded only two form masters Mr. Bowen and Mr. Quick, until it enlisted Mr. Colbeck, Mr. Pratt, Mr. Davidson and Mr. Guillemard, most of the junior form masters took some share in the mathematical teaching. It must have been a difficult task inspiring and unifying their various efforts.

They were not all even professing mathematicians. Quick, for instance, distinguished as a writer on education, was a very slow calculator, so slow that he conscientiously worked out on paper beforehand the questions of mental arithmetic that he addressed to his class.

In its early days the intellectual calibre of the modern side was small. It was entered only through the classical side. The intellectual cream was skimmed and retained on the classical side by almost all tutors.

As the material improved,—such men as F. C. Burkitt, now Norrisian Professor of Divinity at Cambridge, and F. Irwin, Professor of Mathematics at Princeton, U.S., passed through it,—scholarships were won at Cambridge in Mathematics, and high places in the examinations for Woolwich.

That the all-round standard of teaching was high was due to the two men who mostly inspired it, Messrs. Bowen and Bushell.

It is difficult to recall the exact mode of Bushell's influence on his junior colleagues, but it certainly was towards freedom in Mathematics and towards practical applications of them.

He was an active member of the Association for the Improvement of Geometrical Teaching at its origin in 1871, and it was interesting to note, when the revolution in mathematical teaching occurred thirty years later, how closely the demands for reform resembled—even in actual wording—those made at that time. Hints that Bushell gave to his junior colleagues, generally touched with humour, were inspired by such ideas, and impressed them with the readiness of his knowledge and the indefatigable labour which he gave to the most advanced mathematics done in the school.

Bushell had many interests, theology, archaeology, astronomy, etc. Once, thinking to overawe a precocious boy with his interest in the last, he left a fine photograph of the moon, then a novelty, in his study. But when his pupil caught sight of it he sprang to it saying, "How lovely! there's Copernicus and Tycho Brahe!" and Bushell told the story with amusement, seeing that he had missed his aim but gained the fellowship of a kindred spirit. The memory of mathematical meetings at Mr. Bushell's house is one of brisk business, constantly enlivened by a kindly humour.

MATHEMATICS IN THE LYCÉES,

By Miss E. M. Read, B.Sc.

When I heard from our secretary that the Mathematical Association was asking me to read this paper upon my experiences in France, I was very sensible of the honour you were paying me, but very diffident of my power to tell you anything of value, as the result of so short a visit as I was able to make. I was in France during October and November only, and for a week or two of that time the schools were not in working order, as they do not commence until the 1st October. I ask your indulgence for the scantiness of my knowledge, and just do my best to tell you what I can.

France is divided for the purpose of educational administration into nineteen academies, and to gain admission to any of the lessons of the government schools, it is necessary to have a letter of authorisation from the *Recteur* of the academy concerned. I had a notion—unfounded, as it turned out—that French *girls* were not taught very much Mathematics, and so I applied to the *Recteur* of the Paris Academy, almost immediately on my arrival, asking him to allow me to visit boys' *lycées* as well as girls', but the idea of a woman visiting boys' schools seemed quite alien to the French mind, and the permission I sought was not granted me. The nearest I got to the inside of a boys' school in France was the headmaster's private study in a little Catholic school in Tours, and as I did not even succeed in seeing the headmaster there, I cannot set that down as a great victory. Had I had more time, I might have tried harder; as it is, I can only hope that it will not be quite so much of a shock to the *Recteur* the next time he receives a similar request.

As the girls' lycées have been preparing, for the last few years, for the same examinations as the boys', the deprivation did not matter so much as it might otherwise have done, and I was made very welcome in the girls' schools, and given practically *carte blanche* to see and hear anything I cared to ask for.

The lot of the French headmistress seems to me a particularly heavy one. Without exception, in the schools I visited (and I believe the same custom prevails throughout France) she lives in the *lycée* or *école* itself, and her responsibility never ends, so that in a sense, and in a very real sense, I think, her work is never done. It is true that she herself teaches very little, about one lesson a week seems a very fair average, and she has a *surveillante générale*, who must be a great asisistance to her; but I cannot help thinking how much fresher she would be, if she could leave school behind her altogether after school hours are over, and I discovered that she usually shared my view, when I had the opportunity of asking her. On the other hand the *professeurs* (*i.e.* the assistant staff) appear to have a very fairly comfortable time. They teach sixteen hour-lessons a week, and have no other duties at all, except of course any preparation or corrections connected with those lessons. They are not expected to be in school, except during their teaching hours, and many of them find it possible to manage a home and bring up their children satisfactorily outside school hours. It is not surprising perhaps that they do not, as a rule, appear to know their pupils as intimately as we usually do in English schools. I felt that some of the subtler, more vital parts of education were in danger of being neglected in consequence, but I admit I did not stay long enough to become a competent judge, and I am sure that, other things being equal, anything which tends to enable the teacher to live an independent life of her own, and which helps her to look at things from a "non-professorial" angle is to the good.

The school hours are usually about three in the morning and two in the afternoon, but the older girls often have extra classes, and the whole atmosphere of French schools seems rather tense in this respect. The conscientious French secondary school girl, and I fear there are *some* such at any rate, cannot

have very much time for living. I remember asking a class of girls about seventeen years old, how long they spent on their home-lessons. I was having a little chat in English with them, in the presence of their English teacher, and one girl said to me, "This year, I go to bed at ten o'clock, because I was in this class last year; but last year, I could not go to bed before mid-night, because it was my first year in this class." I hope she was not a fair specimen, but certainly a great deal of preparation is expected to be done out of school, and I noticed a girl in whose family I stayed at Rouen, who had very little time at all for anything but her lessons, and she was not in an examination form, nor was she supposed to be working at high pressure at all. Moreover, although the teaching of science in the girls' *lycées* is in general good, and the standard as high, or, probably higher, than in our schools, I saw very little in the way of laboratories for the pupils themselves; and although I am personally of the opinion that very much time is often wasted in English school laboratories, and that the pupil loses there in mental perspective considerably more than she gains in manual dexterity or power of observation, still, there is so much the more time in French schools for sitting still at desks, without change of posture.

Even the *Lycée Jules Ferry*, the Paris *Lycée*, which, I gathered, has the highest reputation for science, does not possess a good laboratory for its pupils, though it has a quite advanced course of scientific and mathematical studies for girls who are intending to be engineers. There is usually plenty of good apparatus for demonstration purposes, and sometimes a pupil instead of the teacher will perform the experiment. I understood that special attention is given to practical manipulation in the normal schools, so that the *teacher*, at all events, shall know how to handle her experiments satisfactorily, and I must say, in most of the lessons I witnessed she managed with conspicuous success.

The domestic science teaching is confined to hygiene and needlework, so that there is no moving about as there would be in a cookery kitchen, and although gymnastics are taught, little or no provision is made for games.

I was very catholic in my tastes, and did not confine my attention to mathematics. I listened to lessons on French history, language and literature, on ancient and foreign literature, on singing, on chemistry, and on physics. Once or twice I tried to hear a lesson on English, but it resolved itself on each occasion into my giving an English conversation lesson. Unluckily, I also missed hearing a lesson on law. This forms part of the regular curriculum for the older girls, but somehow I did not get an opportunity of seeing a lesson given.

As is well known, the French government school is entirely secular. It seemed very strange to me to see all the girls go straight to their lessons, without assembly or prayers, and I could not help feeling that the lack of such a gathering must prevent their having a sense of unity such as we have in our schools here. Every class is given a weekly lesson on some moral subject. The one I heard was on a passage from Charles Wagner, in which is described the ideal character of man.

An hour's discourse on such a subject seemed to me unduly long for both teacher and class; I have known sermons of half that length not seem at all too short. However, the girls had *some* advantage over a church congregation, in that a few of them took some part in the discussion.

In this lesson, as in all others which I saw, I was struck with the method of questioning. The rapid passing from pupil to pupil, which is so often characteristic of an English lesson, finds no place in French teaching, whatever the subject. When once the teacher's attention is directed towards an individual pupil, it is concentrated upon her for about ten minutes. If she fails, another pupil may occasionally assist her, but this is not encouraged. It is her performance, and she is expected to stand or fall by it, and at the conclusion of it

a mark is duly entered against her name. The audience is usually very quiet and attentive while this is in progress, and does not seem to be very much bored, though I noticed some teachers help to ensure their attention by asking one of them afterwards to decide upon the mark to be awarded to the performer.

The elder girls in the *lycée* at Rouen were having one lesson per week in foreign literature. During the course of the year they were expected to cover English, German, Italian and Spanish literature. Another class was doing the same with ancient literature. I fancy they may both suffer later from indigestion.

In the mathematical lessons there seemed to me to be, if I may venture to say so, an undue amount of talking. Very seldom did I see a lesson from which a child could come with a sense of individual achievement. The working of examples by each child separately was left, in a very large measure, for homework, and I felt that the child's burden might have been lightened in this respect—not to mention the teacher's—if some of the work had been done in school hours, as it might have been, had not both teachers and taught been so unfortunately fluent—for fluent they undoubtedly were in general. The children, especially, seemed much less shy and awkward of speech than their English sisters. I am not competent to judge of the grammatical accuracy with which they spoke; my ear is not sufficiently sensitive to the niceties of accent or of phrase-turning, though I noticed the teachers often had to correct for them the genders of their nouns,—and small wonder either.

All the lessons were of an hour's duration, and would have seemed shorter, I think, if less oral work had been done, though I must not omit to mention that many of the teachers made their pupils keep note-books, in which they wrote from dictation the generalised results of the oral discussion—much the same kind of thing that many English science teachers have for their pupils, to render them, for better or worse, independent of printed text.

In some cases, more especially in the schools of Paris, this absence of written class-work was rendered inevitable by the size of the classes concerned. The *lycées* were all overcrowded, and it was no uncommon thing to find a class of forty or even more, and with numbers such as these it is manifestly impossible to supervise written work, even if one sets it. But that this was not the only reason I feel sure, because I saw the same methods at work where the classes were smaller.

Two hours a week are given to mathematics during the first three years, usually three hours during the fourth and fifth. The geometry syllabus of the first year is the use of the ruler, compass, and protractor—polygons, different kinds of triangles and quadrilaterals—the parallelopiped, cube, prism, cylinder, pyramid, cone and sphere—measurements of area and volume. The syllabus in arithmetic is the first four rules, vulgar fractions, decimal numbers, square root (practical rule only) and the metric system.

There is no algebra mentioned here, but, in point of fact, all work with natural numbers is called arithmetic in French text-books, whether such numbers are represented by letters or not. Algebra for them starts with the discussion of directed number. This is a very good thing, I think. Directed number gets very special and careful treatment, *after* the pupil is quite familiar with the use of letters to represent the ordinary well-known signless-number, and when it *is* taught, it is taught properly, and without any, or at least without much, slurring of its difficulties.

With regard to the arithmetic, as we understand the term, it was impossible not to be struck with the enormous advantage they have over us in the matter of units. This is not a question upon which I have ever felt very strongly —such feeling as I have is against the introduction of decimal coinage, etc., into England, but my objections are very mild, and are become even milder

than before, since observing the ease and clarity with which it is possible to establish fundamental principles, when freed of those cumbersome and unscientific British units of weights and measures.

On the other hand, the teaching by rote of square root, before explaining the principle upon which it is based, seems to me an entirely vicious practice —and square root is not a unique example. I often found that arithmetical rules were taught parrot-fashion at first, and the underlying theory deferred. It is true that, later on in their school career, the girls study theory of arithmetic thoroughly, but this does not seem to me to justify its omission at an earlier stage.

I fancy French teachers do not make quite so much of practical geometry, as we have tended to do of late. At all events, I never saw a geometrical instrument in use, either at the board, or at the child's desk. The children drew freehand at the board, whatever was required of them, with reasonable facility, and I do not remember noticing any glaring errors, but, as I did not see a first-year geometry lesson, it may be that the instruments were carefully guarded for the first-year children, and that was why I missed them.

Again, I never saw a lesson in mental arithmetic, although it is part of the regular syllabus.

I mention this, partly because I noticed it at the time, and partly because I have remarked since that Miss Metcalfe of the Board of Education mentions the same thing in her "Notes on the Teaching of Mathematics in French *Lycées* for Girls," included in the Board's publication on Secondary and University Education in France.

She thinks it may have been accidental that she never came across any such instruction, as indeed it may have been in my case too, for what she says is very true—"the children do not seem backward in the ability to carry out the necessary operations both quickly and accurately."

Perhaps it is rather that the teachers incorporate it with the other work, for I remember very well hearing a teacher at S. Quentin teach a class of first-years how to multiply together mentally numbers of two digits.

$$12\times13=(10+2)(10+3)=100+(2+3)10+2\times3=156,$$

and this was in the course of an ordinary lesson, involving the use of the black-board.

No formal geometry is taught till the third year (*i.e.* till about the age of fourteen). The second-year syllabus is devoted to arithmetic (*i.e.* no directed number is introduced). A good deal of work is done on divisibility, on prime numbers, on fractions, and on ratio and proportion, both with natural numbers, and with letters representing them.

It is noteworthy how readily French children seem to take to generalisation. Their minds seem much more apt than ours to discern the underlying principle in the particular case. To them the algebraic statement of a truth is not an added difficulty in their path, as I fear it sometimes is with us—it is obviously useful, and agreeable precision of language. The algebraic shorthand conveys its message spontaneously, and does not need to be laboriously translated into longhand before it can be understood. How much of this is inherent in their mentality, and how much is the result of teaching, I cannot say—but from my own experience of English children, I incline to think it is a racial difference. I saw a second-year class studying the theory of arithmetical long division; they chose their own examples to work on, and I noticed particularly the ease with which they were able to use algebraic symbols to help them in interpreting the arithmetical rule.

In the third year, the study of directed number is commenced, and indeed, I believe little else is undertaken on the algebra side of the syllabus during that year—it is studied thoroughly, and either the French child is very docile, or she really is interested in abstract theoretical mathematics. At all events, little or no jam is offered her with the powder.

The greater part of plane geometry is covered during this year. There was nothing very remarkable about the lessons I saw at this point, but I studied with interest a text-book, written fairly recently on the subject of geometry. It carried one along rapidly, and I believe soundly, by the use of symmetry, both of the *T* and *S* varieties, past the elementary angle, triangle, and parallel work, on to circles, proportion, projection, homothetic figures, and similitude. Pythagoras was proved first by proportion. A proof by a more ordinary area method was given at the end of the book—a much easier one, I thought, than the old I. 47 method. It depended on right-angled triangles only.*

After the third year, some specialisation is allowed, in preparation for the first part of the *Baccalauréat*, which is taken at the end of the fifth year. It is only during the last few years that girls have been entered from the *lycées* for this examination, which was hitherto almost entirely for boys. There are four divisions of this examination. A is chiefly Latin and Greek; B, Latin and Modern Languages; C, Latin and Science; and D, Science and Modern Languages.

In all four divisions a public oral examination of about three-quarters of an hour is given to each candidate on the subjects to which special attention is not directed in the written papers, and to all four, a three-hours' paper on the mother tongue is set. The C's and D's have a four-hour paper on mathematics and physics.

Those pupils who are destined for the C and D divisions do more mathematics than those who are preparing for the A's and B's, and the older programme of the girls' *lycées* has had to be modified to arrange for these newer examination requirements. Cosmography has not lost its place in the schools, although it is not required in the first part of the *Baccalauréat*. Quite a large part of the mathematical time of the fifth year is devoted to it. Cosmography, as any of you who may not know will gather, is a kind of elementary astronomy.

The mathematics required for the examination are a thorough knowledge of plane and solid geometry, some descriptive geometry (which is not algebraic geometry at all, but more solid, and projective), trigonometry and algebra.

The second part of the *Baccalauréat* requires, in addition, a good deal of the theory of arithmetic, conic sections, cosmography, and applied mathematics. It is taken at the end of the sixth year, in some of the large Paris *Lycées*. I saw some girls at work on some quite difficult three-dimensional statics, in the *Lycée Victor Hugo*.

There are two *Écoles Primaires Supérieures* for girls in Paris, a very inadequate number for the capital city.

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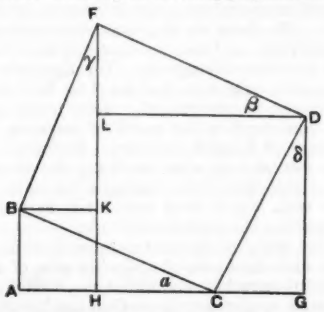
* *Theorem of Pythagoras.* Draw sq. on *BC*, drop perpendicular from *F*, *D*, *B* on *AC*.



The four triangles are equal. Take *FLD*, *FBK* from big square. Add *ABC* and *DCG*]

The scantiness of the provision made for free secondary education was frequently commented upon in my hearing: for the *école primaire supérieure* does, to a considerable extent, supply the type of education which we should probably designate as secondary. The pupils are admitted by competitive examination, and there are no fees. No Latin or Greek is taught, but in some other respects I did not find the instruction differing very much from that given in the *lycées*, except that, as the girls were not as a rule so comfortably circumstanced, they were not able to continue their studies so long. A large number of them were preparing for primary teaching, and were entered at the close of their school career for the *brevet supérieur* with that end in view. I talked for half-an-hour or more in English with a class of elder girls in the *École Edgar-Quinet*, and was very much pleased, and not a little surprised, at the readiness with which they followed me. I also heard lessons in French literature and in physics, which seemed to me, if I may express myself so, more of the secondary than of the primary type.

The mathematics were not so well advanced, though they were being very reasonably taught, as far as they went, and although algebra (as the French understand it) is not prescribed for girl candidates for the *brevet supérieur*, it was certainly taught to some extent. I remember hearing a very good lesson in such a school on directed number.

I do not think I have really very much to say about schools in the devastated areas, except to speak with intense admiration of the brave way in which both teachers and children are carrying on, under most difficult conditions. The *lycée* at Reims was used as an observation post during the war, and the bursar there told me she had stayed throughout the war, nursing and attending to the needs of the soldiers who were stationed there. The *lycée* itself is practically rebuilt again now, and things are fairly normal, as far as that is concerned, but the monotony and dreariness of life in the ruined town must be very difficult to endure. There are no distractions of any kind, and if one seeks to escape from the sad and dreary sights of the town into the country round about, it is more desolate still—all is blasted and deserted.

St. Quentin was considerably worse, for there, there is hardly decent living accommodation, and scarcely a pane of glass in the windows throughout the town. I spent several nights in a little hotel there which had been occupied by Germans during the war. The windows were covered with a sort of grease-proof paper, which shut out most of the light and some of the rain. It rained nearly all the time I was there, and as I was sleeping on the top floor, and there were holes in the roof, I spent a large portion of my nights in moving my possessions around out of harm's way. The staff of the *lycée* lived in much the same fashion, or else in very new houses, without adequate heating arrangements. It was quite impossible to keep dry garments. The moisture from the damp, new walls made them all unfit to wear, and yet they had to be worn all the same.

Some of you may know Mme. Chalmel, who was at Bedford College all last year. She told me how very different it seemed from the luxury with which she was surrounded in England. She was very cheerful, notwithstanding—almost artificially so. In fact they all were. I suppose it was the only way to keep going at all. The *lycée* itself had been very largely destroyed, and was in course of reconstruction during my stay. There were no cloak-rooms or facilities of any kind for either children or staff—they just came into the class-rooms with their outdoor garments on, or put them on the desks beside them. The noise of hammering and sawing was continually in one's ears, and must have been very trying to bear for long.

Notwithstanding all the difficulties, I heard some very good lessons given. It was sad to think we could do so little to help them. What they want is houses, and those we cannot send them, and I fear it will take a great deal of

time before the work of destruction can be cleared away sufficiently to enable the work of reconstruction to progress at any rapid rate.

May I say, in conclusion, that I brought back syllabuses of the general plan of study in the girls' *lycées*, and also the regulations for the *baccalauréat* and the *brevets*, as well as some mathematical text-books; and if I have any information which may by chance be of use to anyone here, I shall be most happy to supply it.

Mr. Twentyman of the Board of Education was my great friend and helper throughout, and he has undertaken to cope with any demands which may be made upon him, as a consequence of my publishing abroad the news of his kindness to me.

Thank you all very much for the most kind way in which you have listened to me.

BOOKS.

Plan d'études et programmes de l'enseignement secondaire des jeunes filles (Librairie Vuibert, 63 Boulevard S. Germain, Paris). 1 franc.

Le nouveau Baccalauréat de l'enseignement secondaire (Librairie Croville Movant, 20 rue de la Sorbonne). 1 f. 50.

Ditto for *Brevet Supérieur* and *Brevet Simple*. 0,75 each.

Cours d'Arithmétique (Vuibert), par L. Tripard.

Algèbre. (Librairie Armand Colin, 103 Boulevard S. Michel, Paris), by Émile Borel.

Trigonométrie, par F. G. M. (Gigord, 15 rue Cassette, Paris).

Leçons de Mécanique Elémentaire à l'usage des élèves des classes de mathématiques A *et* B (P. Appell).

(Gauthier-Villars, 55 Quai des Grands Augustins, Paris.) *Electricity, etc. Cosmography. Chemistry.*

Geometry. André Viéilleford (Garçons), 1e et 2ième cycle, A et B (Hachette, 79 Boulevard S. Germain).

E. M. READ.

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73. Moveable Figures in Books.—*e.g.* Geometry, etc.

Cowley's Perspective, 1766, and Harrington's Science Improved, 1774.

"Mr. Blundevil his Exercises," 7th edn. 1636, has three moveable figures.

Opusculum Geographicum rarum, totius eius negotii rationem, mira industria et brevitate complectens . . . ex diversorum libris et chartis . . . collectum et publicatum, per Joannem Myritium Melitensem, . . . fol. Ingolstadii, 1590. Astronomy, eclipses, etc., illustrated by moveable figures.

Renatus des Cartes de Homine. Figuris et Latinitate donatus a Florentio Schuyl, Lugd. Bat., 4to, 1664. At p. 6 is a heart with two flaps, which lift up to show the inner structure.

Bartisch. Ὀφθαλμοδουλεια, Das ist Augendienst, Dresden, 1583. The anatomy of the eye, illustrated by a series of woodcuts, placed one over the other.

The Kalendar or Ephemerides of Johannes de Monte Regio, Venice, 1482, small 4to.

Four tables at end, the second of which, *Instrumenta veri motus lune*, has two moveable cards attached by a thread through the centre, showing the moon's motions. The fourth *Quadratu borariu generale*, a card or index attached by a moveable two-jointed brass arm, securely fastened to the leaf.

First edition, n.d., supposed Nuremberg in 1473, where second edition in 1474. Another, Venice, 1476.

(Titles above are as given in N. & Q., V. ii. and iii.)

R. C. (N. & Q., V. iii. p. 17) says he possesses a manuscript copy of Raymundi Lullii *Ars Generalis*, 1306, with four astrological figures or tables, one of which has two round moveable pieces of parchment attached, also through the centre by a thread.

REVIEWS.

School Dynamics. By W. G. Borchardt. Part I. Pp. vii + 286 + xix. 6s. Or in two parts, 3s. 6d. each. 1920. (Rivingtons.)

Mr. Borchardt's course Part I. suffices for the demands of the Junior Locals, Part II. including all that is required for the Senior Locals and for the Army, etc. The text is clearly written, the admixture of experiment and theory has evidently been the outcome of much careful thought and of long experience. The author believes in the clarity of thought involved by the expression of results in formulae rather than in words, holding that this outweighs at this stage the advantage gained by statements in words. All teachers will not agree on this point, and the weariness which Mr. Borchardt dreads to see arising from the second of the alternatives, and the confusion which he has found to persist in consequence, may possibly be attributable to the comparative neglect of English as a subject on a modern side. Both the absolute and gravitational systems of units are employed in the formulae dealing with Force; historical stimulus is not forgotten, *e.g.* in the account of the experiments of Galileo; and where methods of approach are lacking in unanimity of support the teacher is left free to adopt his own plan, and the arrangement is such that he can do this with comparative ease. The most gratifying feature about the book is the appearance of rigid dynamics in a course so elementary. In these days when most boys have, or wish to have, ideas about motor cars, and cycles, aeroplanes and gyrostats, there is everything to be said for such an innovation. Graphical methods are carefully set forth and explained, and appropriate stress is laid on the applications of the principle of Energy. The examples are numerical and should excite the interest of every boy of a mechanical turn of mind.

William Done Bushell, of Harrow. Pp. 74. 3s. net. 1919. (Cambridge University Press.)

This little pamphlet contains a short account of Mr. Bushell's life, and of his work as a teacher and as an archaeologist, from the pen of his son, Mr. W. F. Bushell. Canon Glazebrook writes of him as a housemaster, and Father Denys tells the tale of Mr. Bushell's connection with Caldley Island. There are seven illustrations. We make no apology, in this jubilee year of the A.I.G.T. for quoting the following passage:—"Mr. Bushell was an original and enthusiastic member. It is possible that little was accomplished at first, but it did succeed in two great points:—(*a*) in bringing together masters from different schools who might not otherwise have known one another, and (*b*) in sowing the seed of future reform. Most reforms are classed as heresy at first—but the heresy of 1871 became the common-place of twenty years later, and now-a-days would be regarded as antiquated. The Association . . . were by no means a quiescent body; they were not satisfied with turning their attentions to Euclid, but soon extended them so as to cover other branches of Mathematics." The great activity of the Mathematical Association and the dethronement of Euclid during the last ten or fifteen years have overshadowed these early efforts; but it was then that the seed was sown, and Mr. Bushell must be given full credit for his share in the pioneer work. Of the earlier reforms Mr. Bushell was a keen advocate; and when after his day Euclid as such passed away he always hesitated to express a decided opinion as to the situation, feeling that he did not wish to hamper the efforts of the younger men who were then conducting the campaign of reform. He had, however, a great belief in the form and substance of geometrical reasoning, thinking that it was conducive to clearness of expression and thought. And though this is probably true, modern experiment has shown that the intellectual appetite of the dullard at all events can never be satisfied by the formality and stiffness of Euclid.

An Introduction to String Figures. An Amusement for Everybody. By W. W. Rouse Ball. Pp. 39. 2s. 1920. (Heffer & Sons.)

There are few amusements so inexpensive as the making of string figures. With about seven feet of string and this little book, we are told by the author

that "amusement will be forthcoming to while away many a vacant hour." Most of us are familiar from our nursery days with "cat's cradle," but many may be unaware that this and hundreds of similar devices are to be found amusing the inhabitants of many parts of the world, and that of late these games have become a subject of investigation on the part of the ethnologists. Mr. C. F. Jayne and Mr. A. C. Haddon have both published works on string figures, and that by the former contains nearly 900 diagrams. We cordially recommend to our readers Mr. Rouse Ball's Lecture, which was delivered last spring at the Royal Institution, London.

REPORT OF THE SYDNEY BRANCH FOR THE YEAR 1920.

THE Sydney Branch of the Mathematical Association now includes 23 members and 44 associates—total 68. Of the 23 members, two are school libraries. Membership of this kind, whereby an institution or a school has the advantage of the *Mathematical Gazette* forming a part of its library, is to be encouraged.

During the year two meetings were held; at the first the Secretary spoke on "Statistics as applied to Educational Questions"; at the second, which was also the annual meeting, the President, Prof. Carslaw, spoke on "Some Theories of Map Projection." On account of Prof. Carslaw's intended absence from Sydney during 1921, Acting-Professor Wellish was elected President for next year. The Officers for 1921 are: *President*, Acting-Prof. Wellish; *Hon. Treasurer*, C. A. Gale, B.A.; *Joint Hon. Secs.*, Miss Fanny Cohen, M.A., B.Sc., and H. J. Meldrum, B.A., B.Sc.

University of London.

A COURSE OF FOUR PUBLIC LECTURES
ON
"COSMOGONY AND STELLAR EVOLUTION"
WILL BE GIVEN BY
MR. J. H. JEANS, M.A., F.R.S.
(*Secretary of the Royal Society*)

AT KING'S COLLEGE, STRAND, W.C. 2,
On TUESDAYS, May 3, 10, 17 *and* 24, *at Five p.m.*

THESE Lectures, which will be illustrated with Lantern Slides, will give an account of recent observational and theoretical research and be given in non-technical form so as to be intelligible not only to Astronomers, Mathematicians, and Geologists, but also to all acquainted with simple scientific terminology. Syllabus obtainable on application to the undersigned. The Chair at the first Lecture will be taken by Professor J. W. NICHOLSON, F.R.S. (Professor of Mathematics in the University). ADMISSION FREE.

EDWIN DELLER, *Academic Registrar.*

GLASGOW: PRINTED AT THE UNIVERSITY PRESS BY ROBERT MACLEHOSE AND CO. LTD.

BOOKS RECEIVED, CONTENTS OF JOURNALS, ETC.

March, 1921.

A First Course in the Calculus. "Part II. *Trigonometric and Logarithmic Functions of x, etc.*" By W. P. MILNE and G. J. B. WESTCOTT. Pp. xv+181-402+xv-xxxix. 5s. 1920. (Bell & Sons.)

A Course of Modern Analysis. By E. T. WHITTAKER and G. N. WATSON. Pp. 678. 2nd Edit. 21s. 1920. (Camb. Univ. Press.)

Solid Geometry. By J. W. HENSON, B.A. Pp. vii+88. 3s. net. 1920. (Blackie.)

A First Trigonometry. By W. WADDELL and D. K. PICKEN. Pp. vii+78. n.p. 1920. (Melville & Mullen, Melbourne.)

An Elementary Demonstration of Stirling's Approximate Formula for the Value of $\lfloor n$. By G. J. LIDSTONE. Reprinted from *Journ. Instit. Actuaries.* LII. Pp. 102-106. April, 1920. Pp. 5.

Physics: the Elements. By N. R. CAMPBELL. Pp. vii+565. 40s. net. 1920. (Cam. Univ. Press.)

An Introduction to String Figures. An amusement for everybody. By W. W. Rouse Ball. Pp. 38. 2s. 1920. (Heffer, Cambridge.)

The Mathematical Theory of Electricity and Magnetism. By J. H. JEANS. Pp. vii+627. 24s. net. 1920. (Camb. Univ. Press.)

Plane Algebraic Curves. By H. HILTON. Pp. xv+388. 28s. net. 1920. (Clarendon Press.)

The Elements of Statistics. By A. L. BOWLEY. 4th ed. Pp. xi+459. n.p. 1920. (King & Sons.)

Œuvres complètes de Christiaan Huygens. Vol. XIV. 1655-1666. Pp. 557. n.p. 1920. (Nijhoff, La Haye.)

Soaring Flight. A Simple Mechanical Solution of the Problem. By Lt.-Col. R. DE VILLAMIL. Pp. 48. 1s. 6d. net. 1920. (Charles Spon.)

Principes Usuels de Nomographie, avec applications à divers problèmes concernant l'Artillerie et l'Aviation. Conférences faites à la Section technique de l'Artillerie (Fév. 1919). By Lieut.-Col. D'OCAGNE, Chef de la Section de Nomographie. Pp. 67. 4 fr. 50 c. *maj. temp.* 100 %. 1920. (Gauthier-Villars.)

Géométrie et Analyse des Intégrales Doubles. (No. 36, Scientia.) By A. BUHL. Pp. 68. 3 fr. *maj. temp.* 100 %. 1920. (Gauthier-Villars.)

Intermédiaire des Mathématiciens. Edited by C. A. LAISANT. May-June, 1920.

Notes on the Logarithmic Spiral, Golden Section, and the Fibonacci Series. By Prof. R. C. ARCHIBALD. Pp. 12. Reprinted from Jay Hambidge's *Dynamic Symmetry.* 1920. (Yale University Press.)

The Subject Index to Periodicals. 1917-1919. Issued by the Library Association. F. *Education and Child Welfare.* Pp. 87. 4s. net. Jan. 1920. King & Sons.)

An Elementary Text-Book of Trigonometry. By R. W. K. EDWARDS. Pp. xiii +251. 5s. net. 1921. (Harrap.)

American Journal of Mathematics. (Baltimore: Johns Hopkins Press.)

Oct. 1920.

Geometrical Significance of Isothermal Conjugacy of a Net of Curves. Pp. 211-221. E. J. WILCZYNSKI. *Observations weighted according to Order.* Pp. 222-236. P. J. DANIELS. *Some Determinant Expansions.* Pp. 237-242. L. H. RICE. *A General Implicit Function Theorem with an Application to Problems of Relative Minima.* Pp. 243-256. K. W. LAMSON. *On the Laplace-Poisson Mixed Equation.* Pp. 257-277. R. F. BORDEN. *Characteristic Sub-Groups of an Abelian Prime Power Group.* Pp. 278-286. G. A. MILLER.

The American Mathematical Monthly. (Published by the Mathematical Association of America, Lancaster, Pa., U.S.A.)

Dec. 1920.

The Graphical Solution of Spherical Triangles. Pp. 452-460. H. C. BRADLEY. *Questions and Discussions. Geometrical Proofs of the Law of Tangents.* Pp. 465-467. W. V. LOVITT. *Note on the Law of the Mean.* Pp. 467-9. H. H. DOWNING.

Jan. 1921.

Oblique Deviation and Refraction produced by Prisms. Pp. 1-10. H. S. UHLER. *The First Work on Mathematics printed in the New World.* Pp. 10-15. D. E. SMITH. *A Determination of the Curve minimising the Area enclosed by it and its Evolute.* Pp. 15-19. O. DUNKEL. *On some Geometric Methods for Curve Tracing.* Pp. 21-25. R. L. BORGER. *A Figure of Solid Analytic Geometry.* Pp. 26-28. J. W. BRADSHAW.

Feb. 1921.

On the Construction and Modelling of Algebraic Surfaces. Pp. 46-53. A. EMCH. *A Curve of Pursuit.* Pp. 54-61. F. V. MORLEY. *Two Mathematical Shrines in Paris.* Pp. 62-63. [Descartes: Pascal]. D. E. SMITH. Discussion: *On Exact Differentials.* Pp. 66-68. J. W. CAMPBELL. *On the Teaching of Limits in the High Schools.* Pp. 68-70. J. V. MCKELVEY. *Geometric Proof of the Law of Tangents.* Pp. 71-73. C. A. EPPERSON. *Finite Geometries.* Pp. 85-87. U. G. MITCHELL.

Annals of Mathematics. (Published by Princeton University, Lancaster, Pa., U.S.A.)

Dec. 1920.

The Mean of a Functional of Arbitrary Elements. Pp. 66-72. N. WIENER. *On Certain Determinants associated with Transformations employed in Thermodynamics.* Pp. 73-85. J. F. TREVOR. *The Permanent Gravitational Field in the Einstein Theory.* Pp. 86-94. L. P. EISENHART. *On the Structure of Finite Continuous Groups with a Finite Number of Exceptional Infinitesimal Transformations.* Pp. 95-100. S. D. ZELDIN. *Conformal Mapping of a Family of Real Conics upon Another.* Pp. 101-127. T. H. GRONWALL. *On the Location of the Roots of the Derivative of a Polynomial.* Pp. 128-144. J. L. WALSH.

British Astronomical Journal.

Four Geometrical Solutions (of the Problem of determining the hour angle of rising or setting, altitude, and azimuth) by J. SHORT. June, 1920. Pp. 281-4. *Remarks on Mr. Short's Solutions.* Pp. 76-81. The Rev. M. DAVIDSON.

Bulletin of the American Mathematical Society. (Published by the Society, Lancaster, Pa., U.S.A.)

Nov. 1920.

Note on Velocity Systems in Curved Space of n Dimensions. Pp. 71-77. J. LIPKA. *Augustus De Morgan on Divergent Series.* Pp. 77-81. F. CAJORI.

Dec. 1920.

Note on a Method of Proof in the Theory of Fourier's Series. Pp. 108-110. D. JACKSON. *On Implicit Functions.* Pp. 111-113. F. H. MURRAY. *On a Pencil of Nodal Cubics* (Second Paper). Pp. 114-115. N. ALTSHILLER-COURT. *Hausdorff's Grundzüge der Mengenlehre.* Pp. 116-129. H. BLUMBERG.

Jan. 1921.

An Image in Four-Dimensional Lattice Space of the Theory of the Elliptic Theta Functions. Pp. 153-160. E. T. BELL. *Note on the Median of a Set of Numbers.* Pp. 160-164. DUNHAM JACKSON. *Note on Closure of Orthogonal Sets.* Pp. 165-169. O. D. KELLOGG. *The Mathematical Work of T. Jan Stieltjes.* Pp. 170-178. R. D. CARMICHAEL.

Bulletin of the Calcutta Mathematical Society.

Dec. 1920.

Evaluation of the Product Matrix in a Commutantal Product of Simple Matrices having given Nullities. Pp. 105-150. C. E. CULLIS. *A Note on the Construction by Points of a Trinodal Quartic.* Pp. 151-156. G. MUKERJEE. *On Liquid Motion inside Rotating Arcs of Three and Four Confocal Parabolas.* Pp. 157-172. N. M. BASU. *Investigations on Ripple Motion.* Pp. 173-178. J. C. K. RAO. *Note on the Product of any Number of Legendre Functions of Different Degrees.* Pp. 179-184. S. BANERJI.

Gazeta Matematica. (Göbl. Bucuvesti.)

Nov. and Dec. 1920.

The Journal of the Indian Mathematical Society. (S. Varadachari, Madras.)

June, 1920.

Obituaries of S. Ramanujan and J. G. Darboux.

Aug. 1920.

Double Points and Lines. Pp. 121-135. M. B. RAO. *An Extended Case of Hermite's Theorem.* Pp. 136-7. G. S. M. *Aberration in Latitude and Longitude.* Pp. 137-138. C. KRISHNAMACHARI.

L'Enseignement Mathématique. (Gauthier-Villars, Paris.)

Dec. 1920.

Sur un théorème de cinématique. Pp. 163-169. C. CAILLER. *Généralisation des coordonnées polaires. Applications.* Pp. 170-175. E. JABLONSKI. *Sur le système de nombres bi-complexes.* Pp. 175-183. L. G. DU PASQUIER. *Développement d'une puissance quelconque entière et positive de cos x ou de sin x en fonction linéaire des cos et sin de multiples de x.* Pp. 184-187. E. BARBETTE. *Analyse indéterminée du p^{me} degré sur les sommes de puissances égales des nombres.* Pp. 188-191.

Nieuw Archief voor Wiskunde. Tweede Reeks. Deel XIII. 1920. (Delsman, Amsterdam.)

Die Zahlensysteme der geometrischen Grössen. II. Pp. 240-266. A. A. SCHOUTEN. *Over de regelvlakken van den vierden Graad.* Pp. 267-277. E. J. DIJKSTERHUIS. *Het Nim-spel en uitbreidingen daarvan.* Pp. 278-287. F. SCHUH and P. J. H. BAUDET. *Over twee stellingen van Caratheodory.* Pp. 288-291. J. WOLFF. *Zwei integralgleichungen.* Pp. 292-294. L. CRIJNS. *Het beweijs van de stelling van Pohlke.* Pp. 295-297. C. VAN ALLER. *Over twee formules van Castelnuovo.* Pp. 298-299. J. WOLFF. *Elementair bewijs van een bekende stelling betreffende de priemgetallen, die een rekenkundige reeks vormen.* Pp. 300-301. F. SCHUH. *Over het gebruik van een V_4^2 van R_5 in de metrische Lijnenmeetkunde.* Pp. 302-308. E. J. DIJKSTERHUIS. *Oneigenlijke integratie van een reeks of van een oneigenlijke integraal, met toepassing op de Gammafunctie.* Pp. 309-313. F. SCHUH. *On the Equilibrium of a System of n Particles of Equal Mass, placed on the Inner Surface of a Sphere and mutually repelling each other according to the m^{th} power of the Distance.* Pp. 314-315. A. G. KERKHOVEN-WYTHOFF. *Een Rekenkundig Spel.* Pp. 316-323. *Evaluation of Some Integrals.* Pp. 324-333. T. HAYASHI.

Nouvelles Annales de Mathématiques. (Gauthier-Villars, Paris.)

Sept. 1920.

Surfaces de translation applicables l'une sur l'autre (suite). Pp. 321-341. B. GAMBIER. *Deux notes de géométrie vectorielle.* Pp. 341-347. R. GARNIER. *Un théorème générale sur les complexes.* Pp. 347-355. CL. SERVAIS. *Démonstration géométrique du théorème de Liouville sur le groupe isogonal de transformation dans l'espace.* Pp. 356-358. A. LÉVÊQUE.

Oct. 1920.

Surfaces de translation, etc. (fin). Pp. 361-372. B. GAMBIER. *Sur une classe d'équations différentielles qui admettent des intégrales singulières.* Pp. 372-395. E. GOURSAT. *Sur des systèmes articulés.* Pp. 395-400. R. BRICARD.

Nov. 1920.

Étude des surfaces de translation de Sophus Lie. Pp. 401-423. B. GAMBIER. *Sur la cubique à point double.* Pp. 424-435. N. ALTSHILLER-COURT.

Dec. 1920.

Étude des surfaces de translation de Sophus Lie (concluded). Pp. 454-479. B. GAMBIER. *Sur certaines relations qui existent entre l'epicycloïde et l'hypocycloïde à trois rebroussements.* Pp. 479-484. J. A. MOREN. *Considérations sur le frottement de glissement.* Pp. 485-496. ET. DELASSUS. *Note géométrique sur une généralisation du théorème de composition de vitesses et le théorème de Coriolis.* Pp. 496-501. L. POMEY.

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